2P06

A simple method to evaluate the uncertainty of magnetotelluric foward modeling for practical three-dimensional conductivity structure models

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Introduction

How well an electrical conductivity structure model explains the observed MT responses? Root mean squared (RMS) misfit is an popular way for the evaluation.

$$\mathrm{RMS}_1 = \sqrt{\frac{1}{2N}\sum_{i=1}^{N}\frac{\left|Z_i^{\mathrm{obs}} - Z_i^{\mathrm{syn}}\right|^2}{\left(\varepsilon_i^{\mathrm{obs}}\right)^2}}$$

Z_i^{obs} : ObservedMT response
Z_i^{syn} : Synthesized MT response
$\varepsilon_i^{\mathrm{obs}}$: Standard error of Z_i^{obs}

RMS_1 implicitly assumes that the MT responses are more accurately synthesized than the observational errors. However, this assumption is not always valid.

The uncertainty of the forward calculation may be separated into two components:

Bias component : systematical offset from the true value
Random component : random fluctuations to the true value (← this study focuses on)

In the procedures of the forward modeling, selection of the coordinate system (direction of *x*-axis) may be regarded as a random process because the selection is arbitrary and the synthesized MT responses should be ideally identical irrespective of the coordinate system.

Method

Step 1. Conduct forward modeling in several (j=1,2, ..., M) different coordinate systems.
Step 2. Rotate the synthesized MT responses to a preferred coordinate system, e.g., (x_s, y_s).
Step 3. Evaluate the mean (μ), standard deviation (σ), and coefficient of variation (CV) of the M synthesized MT responses.

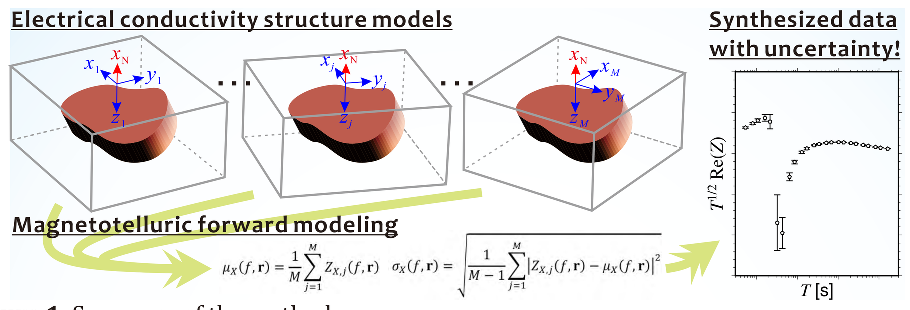

Figure 1. Summary of the method.

Merits : Simple, ease to implement, and broad utility (Applicable to any structure model and numerical algorithm).
Demerit : Time consuming

Applications

3D topography over 1D mantle structure is considered for 10 different coordinate systems. Seafloor MT responses are synthesized using FS3D and two-stage forward modeling algorithm (Baba & Seama, 2002; Baba et al., 2013).

Example 1. Northwestern Pacific – Effect of the regional topography is significant.
Example 2. Southern Atlantic – Effect of the local topography is significant.

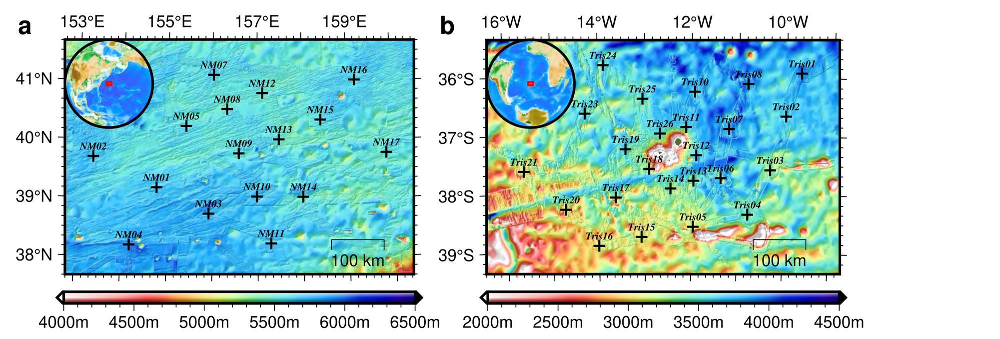

Figure 2. Maps for the seafloor MT observation arrays in **a** NW Pacific and **b** S Atlantic. Color indicates the bathymetry. The location of the array in the globe is indicated by a red rectangle in the inset map.

Example 1

j	θ_j
1	-55°
2	-47°
3	-13°
4	-6°
5	0°
6	15°
7	18°
8	42°
9	66°
10	74°

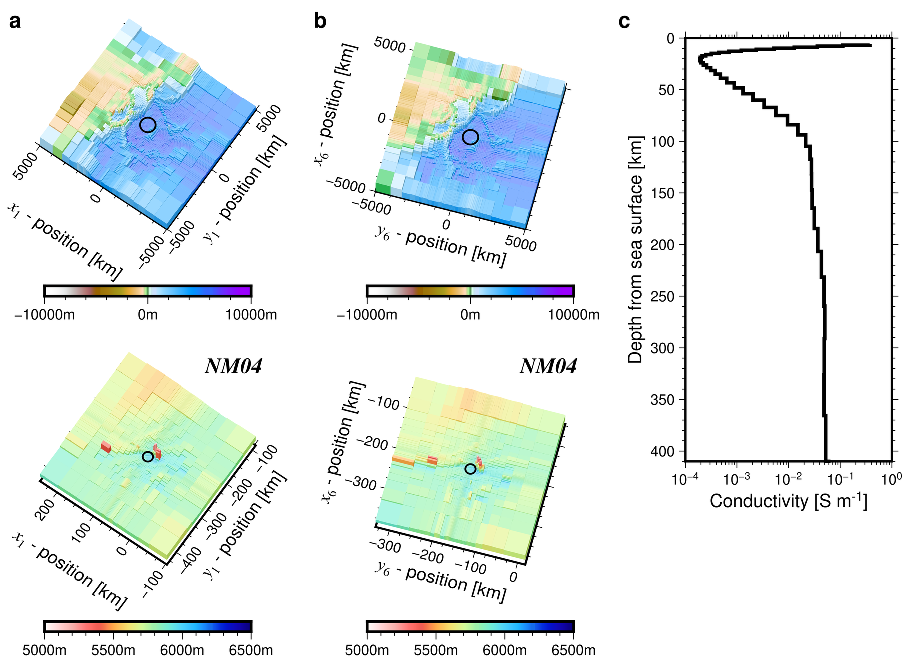

Figure 3. Topography and 1D mantle structure models for NW Pacific. **a** Regional large-scale topography model (top) and local small-scale topography model for a site NM04 (bottom), discretized in the (x_1, y_1) coordinate system (θ_1=-55°). North is up. Color indicates the bathymetry. The observation array or site locates the center of the regional or local model as indicated by a circle. **b** Same as **a** but in the (x_6, y_6) coordinate system (θ_6=15°). **c** 1D electrical conductivity structure model after Baba et al. (2017b).

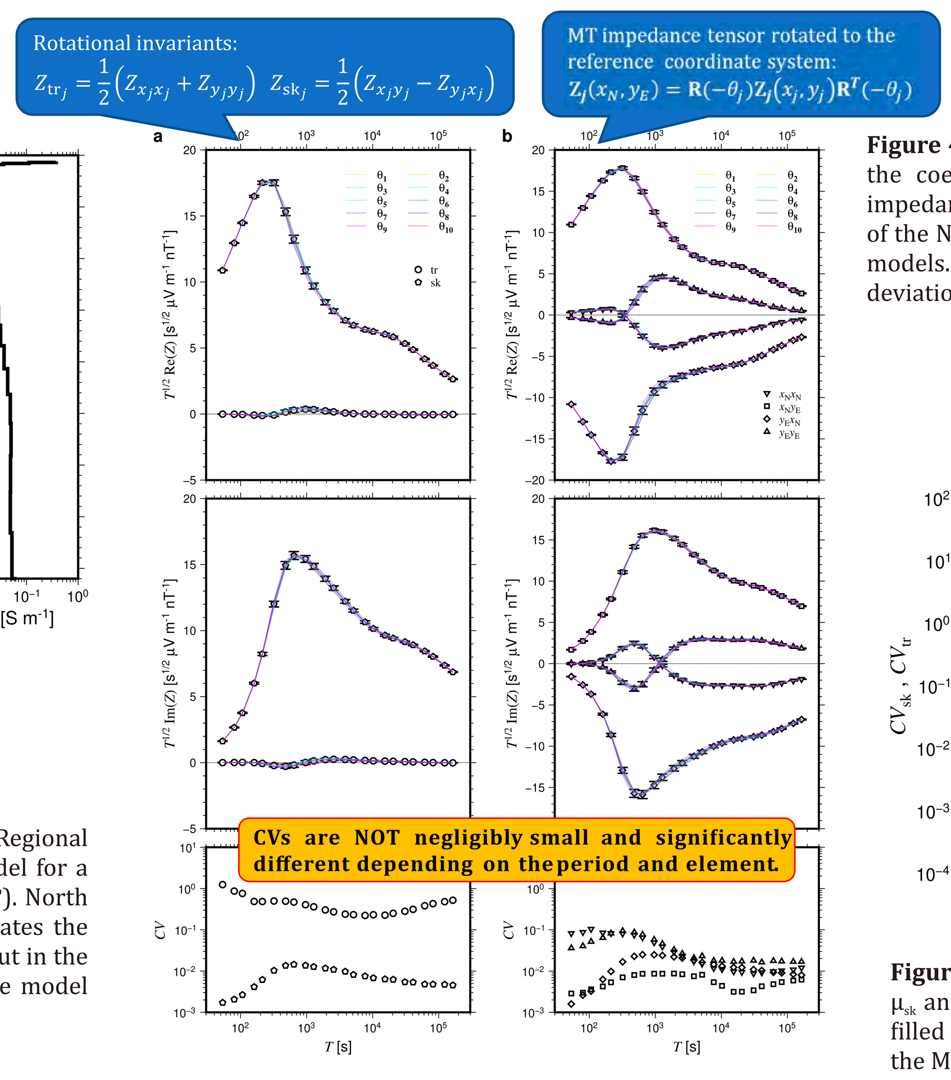

Figure 4. MT sounding curves scaled by the square root of the period and the coefficient of variation of **a** the rotational invariants and **b** the impedance elements rotated to the (x_s, y_s) coordinate system for site NM04 of the NW Pacific models. Colored lines are the sounding curves for the 10 models. Symbol with error bar indicates the mean and the standard deviation of the 10 samples.

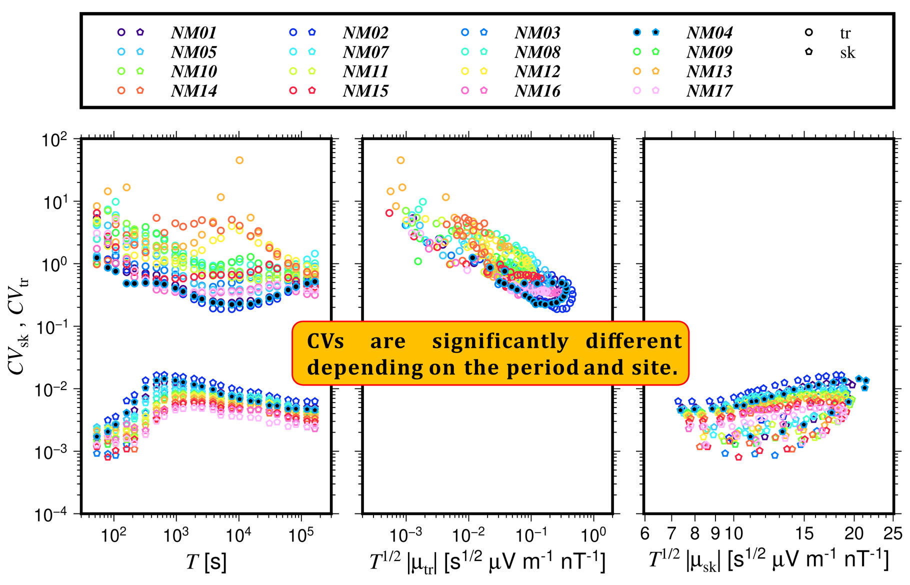

Figure 5. Scatter plots of $\mathrm{CV}_{\mathrm{sk}}$ and $\mathrm{CV}_{\mathrm{tr}}$ with respect to the period and magnitude of μ_{sk} and μ_{tr} scaled by square root of the period for the NW Pacific models. Symbols filled with black indicate the site that the sounding curves and the trajectories of the MT response to the azimuth are demonstrated in Figs. 4 and 6, respectively.

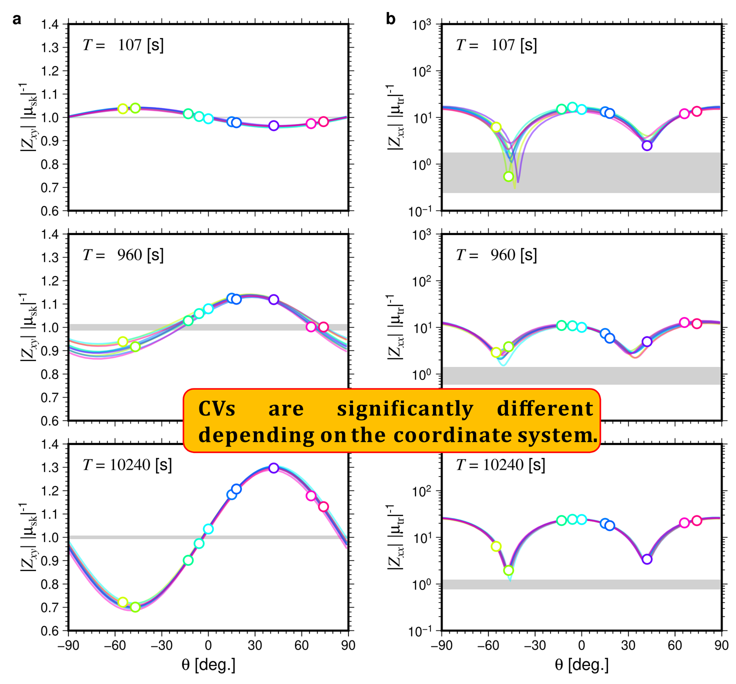

Figure 6. Trajectories (lines) of synthesized **a** Z_{xy} and **b** Z_{xx} (circles) with respect to the azimuth for selected three periods at site NM04 of the NW Pacific models. The values are normalized by $|\mu_{\mathrm{sk}}|$ or $|\mu_{\mathrm{tr}}|$. Gray shades indicate the range of $1\pm\mathrm{CV}_{\mathrm{sk}}$ or $1\pm\mathrm{CV}_{\mathrm{tr}}$. Colors correspond to those in Fig. 4.

Example 2

j	θ_j
1	-79°
2	-61°
3	-48°
4	-20°
5	-5°
6	0°
7	13°
8	34°
9	55°
10	62°

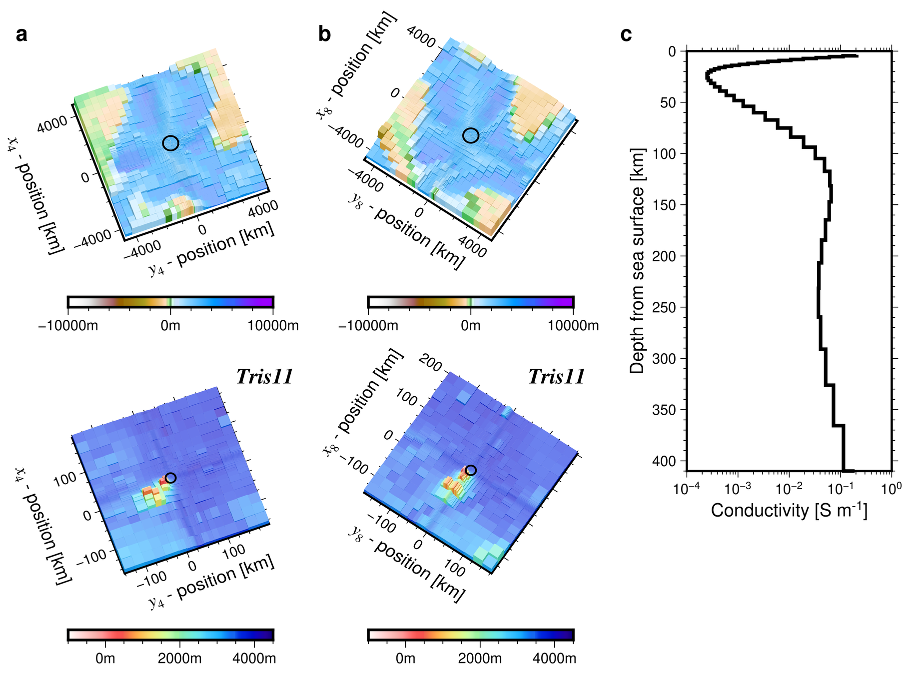

Figure 7. Topography and 1D mantle structure models for S Atlantic. **a** Regional large-scale topography model (top) and local small-scale topography model for site Tris11 (bottom), discretized in the (x_4, y_4) coordinate system (θ_4=-20°). North is up. Color indicates the bathymetry. The observation array or site locates the center of the regional or local model as indicated by a circle. **b** Same as **a** but in the (x_8, y_8) coordinate system (θ_8=34°). **c** 1D electrical conductivity structure model after Baba et al. (2017a).

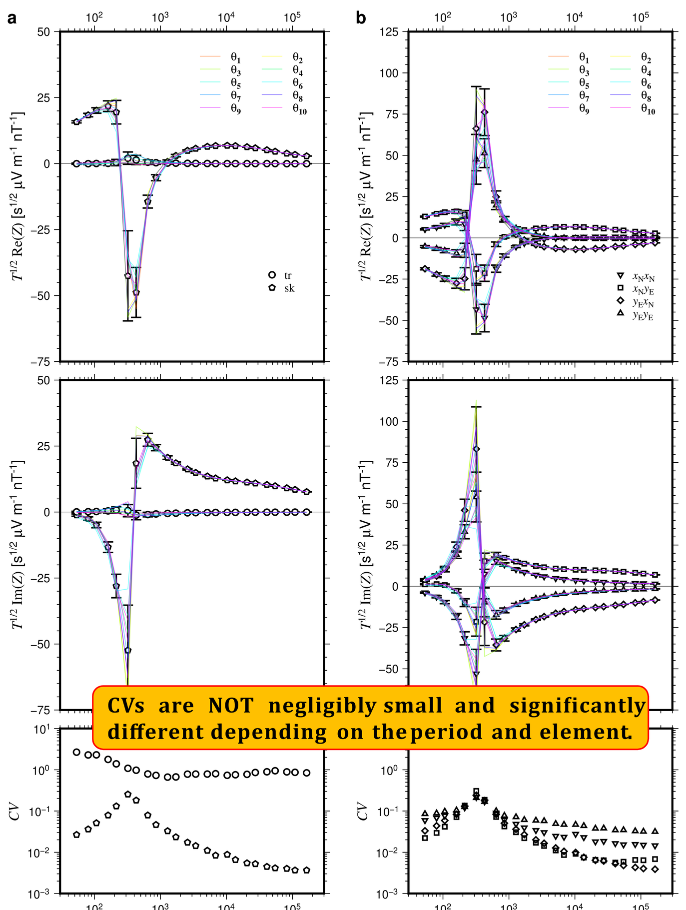

Figure 8. MT sounding curves scaled by the square root of the period and the coefficient of variation of **a** the rotational invariants and **b** the impedance elements rotated to the (x_s, y_s) coordinate system for site Tris11 of the S Atlantic models. Colored lines are the sounding curves for the 10 models. Symbol with error bar indicates the mean and the standard deviation of the 10 samples.

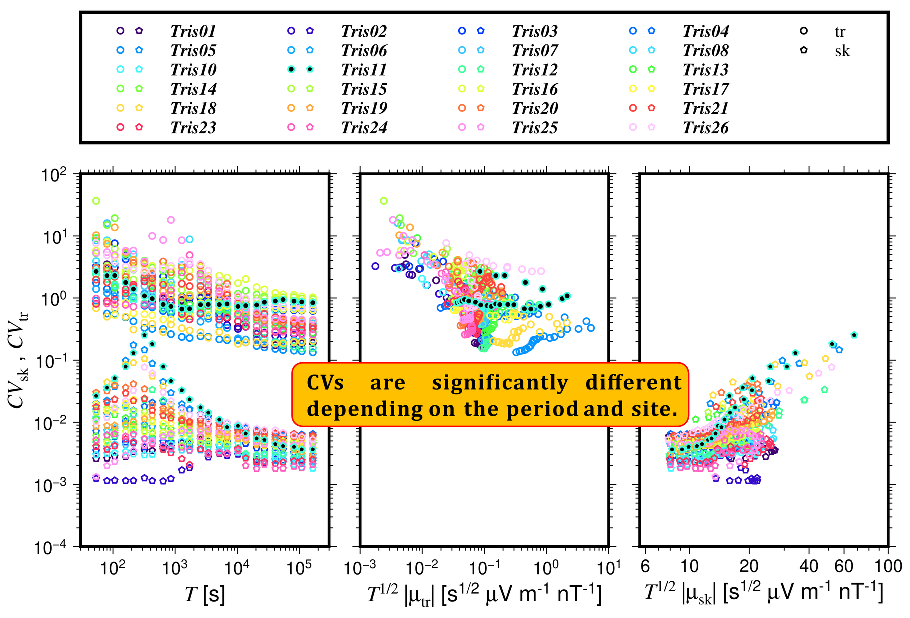

Figure 9. Scatter plots of $\mathrm{CV}_{\mathrm{sk}}$ and $\mathrm{CV}_{\mathrm{tr}}$ with respect to the period and magnitude of μ_{sk} and μ_{tr} scaled by square root of the period for the S Atlantic models. Symbols filled with black indicate the site that the sounding curves and the trajectories of the MT response to the azimuth are demonstrated in Figs. 8 and 10, respectively.

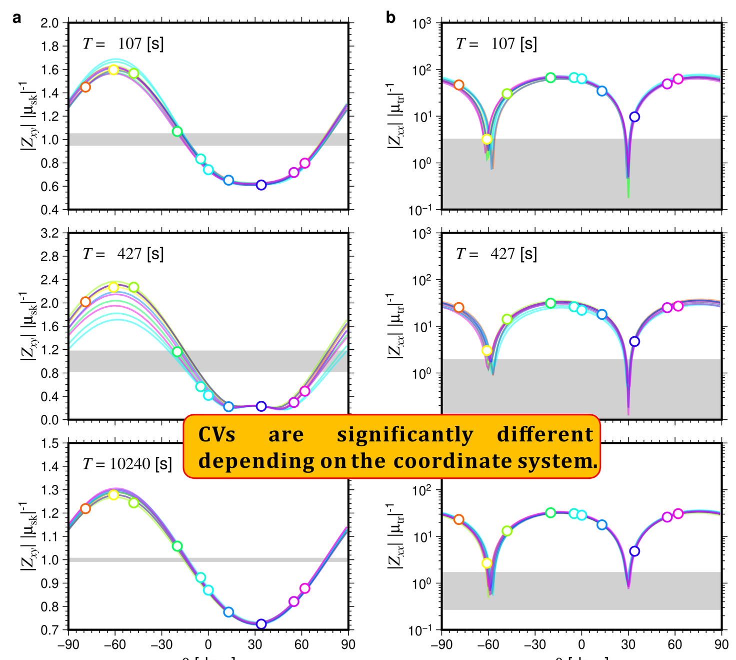

Figure 10. Trajectories (lines) of synthesized **a** Z_{xy} and **b** Z_{xx} (circles) with respect to the azimuth for selected three periods at site Tris11 of the S Atlantic models. The values are normalized by $|\mu_{\mathrm{sk}}|$ or $|\mu_{\mathrm{tr}}|$. Gray shades indicate the range of $1\pm\mathrm{CV}_{\mathrm{sk}}$ or $1\pm\mathrm{CV}_{\mathrm{tr}}$. Colors correspond to those in Fig. 8.

Discussion

A new RMS evaluating the data misfit

$$\mathrm{RMS}_2 = \sqrt{\frac{1}{2N}\sum_{i=1}^{N}\frac{\left|Z_i^{\mathrm{obs}} - \mu_i\right|^2}{\left(\varepsilon_i^{\mathrm{obs}}\right)^2 + \left(\varepsilon_i^{\mathrm{syn}}\right)^2}}$$

Standard error of synthesized responses $\varepsilon_i^{\mathrm{syn}} = \frac{\sigma_i^{\mathrm{syn}}}{\sqrt{M}}$

By using RMS_2 in inversion analysis, overfitting because of ignoring the uncertainty of the forward calculation can be avoided.

Applying epsilon to the normalization works similarly with error floor in terms of avoiding the overfitting problem. The advantage in use of the new normalization is that the evidence and meaning are clear for every data point.

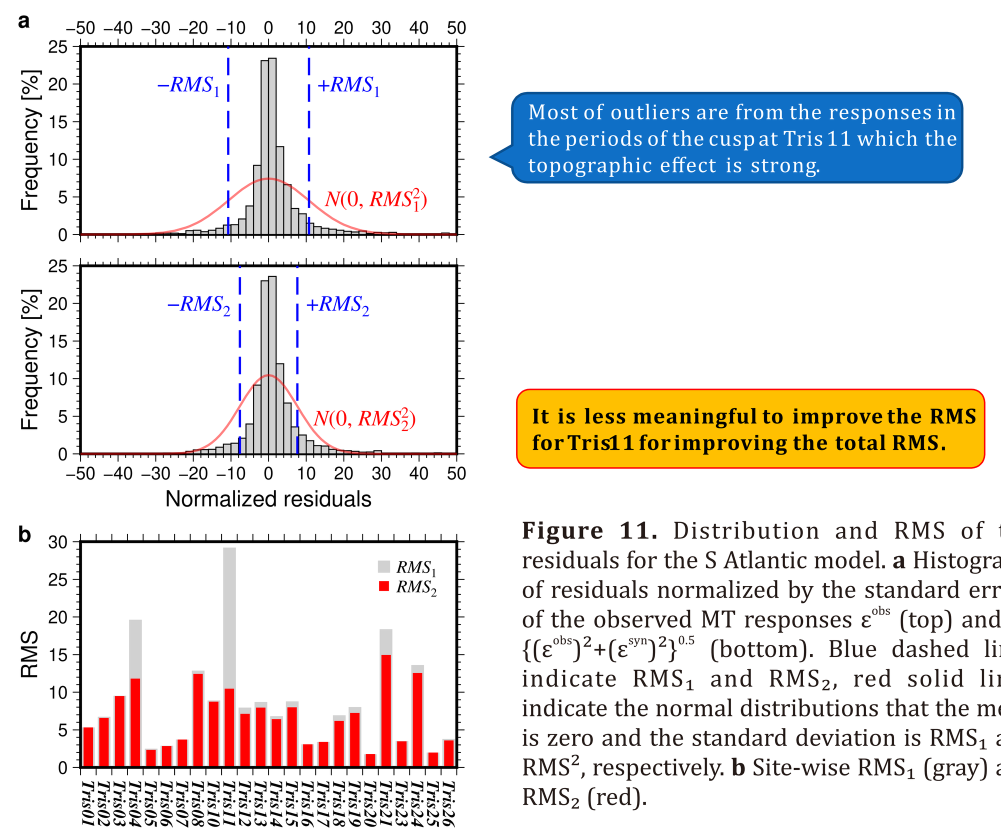

Figure 11. Distribution and RMS of the residuals for the S Atlantic model. **a** Histograms of residuals normalized by the standard errors of the observed MT responses $\varepsilon^{\mathrm{obs}}$ (top) and by $\{(\varepsilon^{\mathrm{obs}})^2+(\varepsilon^{\mathrm{syn}})^2\}^{0.5}$ (bottom). Blue dashed lines indicate RMS_1 and RMS_2, red solid lines indicate the normal distributions that the mean is zero and the standard deviation is RMS_1 and RMS^2, respectively. **b** Site-wise RMS_1 (gray) and RMS_2 (red).

Further merit

The proposed method is also useful for nvestigating of the uncertainty depending on the mesh design. One of the supposed case is to test whether known structures, such as topography, bathymetry, and/or geological setting, are appropriately incorporated into a mesh. Here, I tested the topographic effect modeling in terms of different mesh dimensions for site Tris11 in the S Atlantic.

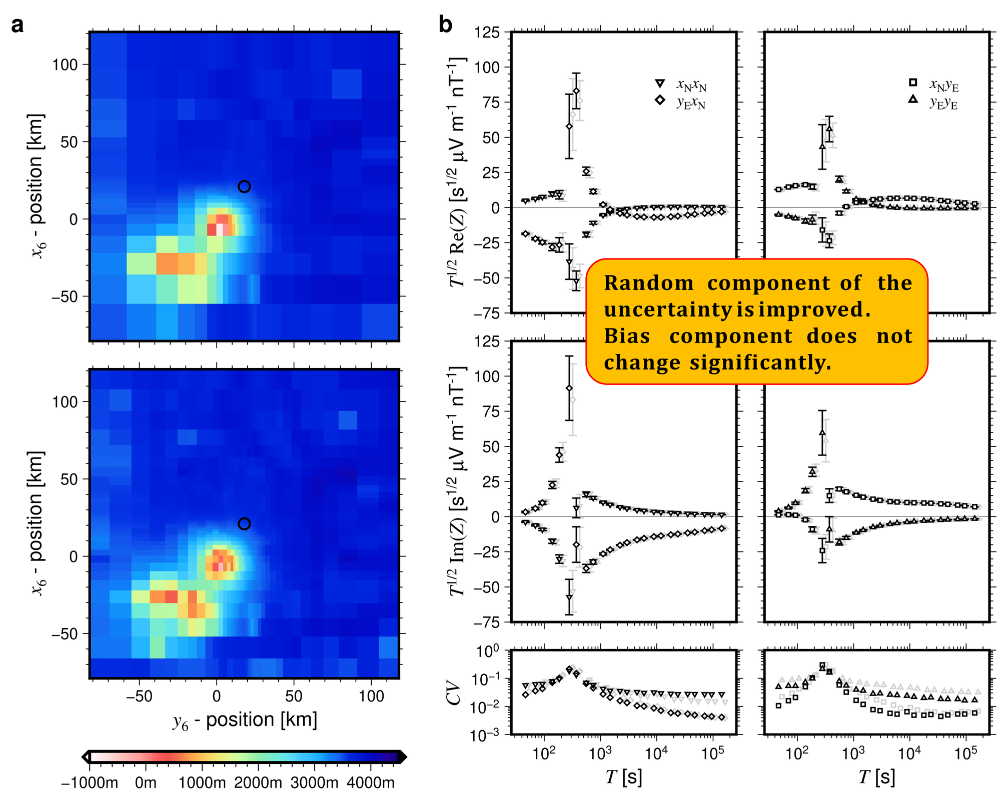

Figure 12. a Local topography models for site Tris11 in S Atlantic discretized with relatively coarser (top) and finer (bottom) meshes in (x_6, y_6) coordinate system. **b** Synthesized MT response for site Tris11 from the models in 10 different coordinate systems (top and middle) and the coefficient of variation of each impedance tensor element (bottom). Symbols are the mean of the MT responses rotated to the (x_s, y_s) coordinate system and error bars are the standard deviations. Black and gray colors indicate those obtained from the finer and coarser mesh models, respectively. The gray symbols are slightly shifted to the right for visibility.

Conclusions

A simple new method is proposed to evaluate the random component of the uncertainty of MT forward modeling to practical 3D conductivity structure models in a Cartesian coordinate system.

The advantages of this method include its easy implementation and its applicability to any 3D structure models and numerical modeling algorithms. By contrast, the disadvantage is that it is time consuming to conduct forward calculations several times.

The uncertainty of the forward calculation should not be neglected, but should be considered for each element, period, and site to quantitatively evaluate how well a given model explains the data.

In this context, a new RMS is proposed in which the residuals are normalized by both the standard errors of the observed and synthesized MT responses.

This method is also useful for testing the appropriate selection of the coordinate system and mesh design.

The trajectory of the MT impedance element to rotation in the horizontal coordinate which will help to find a coordinate system in which the uncertainty is relatively small.

A comparison of the mean and standard deviation between different mesh designs enabled the evaluation of the difference in the stability of the forward calculations and in the bias.

Reference

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