The use of L-curve criteria in non-linear inverse problem

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SUMMARY

Since the ill-posed nature intrinsically exists in almost all kinds of geophysical inverse problems, the regularization that stabilizes the inversion process plays a crucial role in the current deterministic inversion framework. One important issue still open question is which philosophy we should adhere to and how to find the 'optimal' trade-off parameter (λ) to balance data fitting and regularization. Currently, there exists the data-fitting-oriented discrepancy principle, represented as OCCAM, which determines the trade-off parameter by performing a line search and selecting the one that yields a model with a data misfit close to the target. This approach is popular within the EM community. However, it carries a risk of over-fitting when the data uncertainty is not accurately estimated. Another well-known approach is the cooling approach. Its underlying philosophy seems very geologically meaningful, i.e., a reasonable inversion procedure is to gradually add detailed structures to regional ones. However, when one really starts his/her inversion homework, this philosophy can sometimes become a prejudice, as all the settings — such as initial λ, the decreasing rate of λ, and data decreasing threshold — are subjective judgments lacking a reasonable quantitative explanation. Here, we advocate for the L-curve criteria. This approach is theoretically sound and well-validated in mathematics. However, it's important to note that the L-curve was originally designed for linear inverse problems in which the quality (e.g. distribution of singular values) of the forward matrix is fixed. Using it in nonlinear inversion scenarios, such as 3D MT inversion, the nonlinearity of MT should be considered. In this work, through three simulated cases and the USArray data application, we will show (1) the potential risks when the nonlinearity of inversion is not considered, (2) the merits of L-curve criteria when it is correctly used, (3) the underlying connection between L-curve and cooling in nonlinear problems.

Keywords: ill-posed nonlinear inversion, magnetotellurics, trade-off parameter, L-curve criteria