Strength of electromagnetic fields in multilayer spherical media based on generalized reflection coefficients

Ya Gao[1], Qing-Yun Di[1],Zhong-Xing Wang[1],Zhe Yun[1] ,Yi-Lang Zhang[1] ,Wen-Hua Zhang[1] ,Xiong Yin [1] and Xing-Han Li[1]
[1]CAS Engineering Laboratory for Deep Resources Equipment and Technology, Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing 100029, China., gaoya@mail.iggcas.ac.cn

Summary

Electromagnetic fields under large-scale modeling have become important research in the field of geophysical electromagnetic sounding. Although most of our previous studies have been based on horizontal layered models, the curvature of the earth should be considered as the scale of the investigation increases. Therefore, we use the full wave method to rederive the formula in spherical coordinates by introducing the concepts of transmission and reflection, considering the existence of displacement current in the air layer and the coexistence of polarization and conductivity of underground media. Finally, the propagation characteristics of electromagnetic fields of different frequencies are analyzed. The necessity of considering the curvature of the earth for large-scale modeling is illustrated by comparing the results with those under the planar model.

Keywords: multilayer spherical medium; electromagnetic fields; generalized reflection coefficients

Introduction

When the overall study area for geophysical electromagnetic methods spans thousands of kilometers, the horizontal distance in model space will be much larger than the Earth's radius. These continental-scale investigations of EM methods thus pose a new challenge: are traditional Cartesian coordinates still applicable when the curvature of the Earth is not negligible? (Han et al.,2020).

The propagation of waves in a spherical earth has been extensively studied by many reputed researchers. In the 1960s, Wait carried out fundamental theoretical studies of very low frequency (3-30 kHz) wave propagation in radio engineering in spherical coordinates (Wait,1977). Watson obtained the scattered electromagnetic field at the Earth's surface and used the residual theorem to change the form of the sequence sum of the electromagnetic field to an integral form (1981). Houdzoumis (2000) and Barrick (1999) investigated the form of the electromagnetic field in the ideal conductor layer of the “Earth-ionosphere” waveguide and further refined Wait's calculations. Pan (2013) and Wang (2011) investigated the propagation characteristics of SLF/VLF electromagnetic fields in a three-layer spherical waveguide in the Earth's ionosphere under non-ideal electric conductors.

Since these studies are mainly used for electromagnetic field communication, the ionosphere is usually regarded as an impedance boundary with a certain conductivity. More attention is paid to the scattering effect of the Earth's spherical surface under plane-wave incidence and the propagation properties of the field in the cavity, without analyzing in detail the effect of the electrically stratified structure of the Earth's interior on the field.

We need to consider the layered structure of the Earth's interior. In order to make the structure of the formula more flexible, we borrow the idea of generalized reflection coefficients and derive the analytical formula for the electromagnetic field under the multilayer spherical model. By using the boundary condition of tangential continuity of the electromagnetic field, we give the definite solution under this problem. Finally we compare the results in the spherical coordinate system with the horizontal layered model to illustrate the effect of the curvature of the earth on the electromagnetic field.

Methods

Model

As shown in Figure 1, we build a multilayer spherical medium in which the current source and the equivalent magnetic current source are located in the excitation layer. The origin of the coordinate system is chosen as the center of the earth, i.e., the innermost part of the earth represents the first layer, and the number of layers increases outwardly.

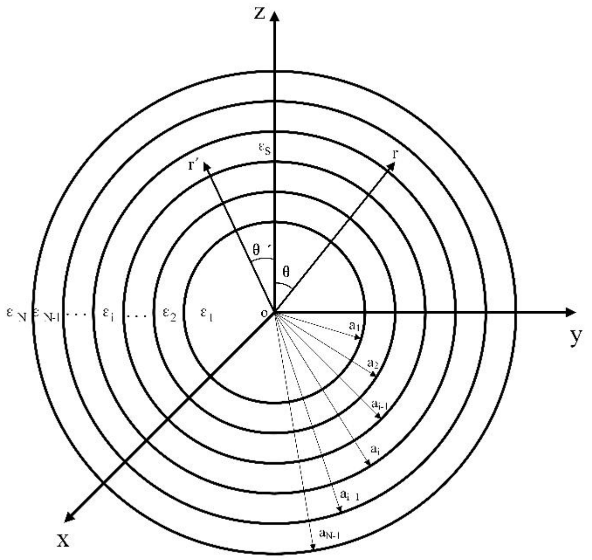

Figure. 1 model of multilayer spherical medium

The parameter r' is the location of the emission source， r is the location of the receiving point. a_i is the radius of the ith spherical layer.

Formula

By using the addition theorem of spherical harmonic function and spherical Bessel function, the field generated by a power source in uniform unbounded space is expanded, and the Debye potential expression with source layer is obtained by the expression of radial electric field and magnetic field：

$$\pi_e = ikIl\boldsymbol{\alpha}\bullet\nabla'\times\nabla'\times\boldsymbol{r}'\sum_{n=0}^{\infty} j_n(kr<)h_n^{(1)}(kr>)\frac{A_n(\theta,\varphi,\theta',\varphi')}{n(\mathrm{n}+1)}$$

$$\pi_m = -\omega\mu kIl\boldsymbol{\alpha}\bullet\nabla'\times\boldsymbol{r}'\sum_{n=0}^{\infty} j_n(kr<)h_n^{(1)}(kr>)\frac{A_n(\theta,\varphi,\theta',\varphi')}{n(\mathrm{n}+1)} \quad (1)$$

If we place the source in the j layer, there will be additional reflected waves

$$\pi = D'\sum_{n=0}^{\infty}[j_n(k_j r<)h_n^{(1)}(k_j r>) + a_{jn}h_n^{(1)}(k_j r) + \mathrm{b}_{jn}\, j_n(k_j r)]\frac{A_n(\theta,\varphi,\theta',\varphi')}{n(\mathrm{n}+1)} \quad (2)$$

According to the boundary condition of continuity of tangential electromagnetic field, the value $\boldsymbol{a}$ and $\boldsymbol{b}$ can be obtained. Finally, we derive the recurrence formula of multilayer spherical media

$$\pi = D_j'\sum_{n=0}^{\infty}F_n(\mathrm{r},\mathrm{r}')\frac{A_n(\theta,\varphi,\theta',\varphi')}{n(\mathrm{n}+1)} \quad (3)$$

When $r\in$ region j and $r'\in$ region j

$$F_n(\mathrm{r},\mathrm{r}')=[h_n^{(1)}(k_j r>) + \widetilde{R_{j,j+1}}j_n(\mathrm{k}_j\mathrm{r}>)][j_n(k_j r<) + \widetilde{R_{j,j-1}}\mathrm{h}_n^{(1)}(\mathrm{k}_j\mathrm{r}<)]\widetilde{M_j} \quad (4)$$

When $r\in$ region i （$i>j$）

$$F_n(\mathrm{r},\mathrm{r}')=[j_n(k_j\,\mathrm{r}') + \widetilde{R_{j,j-1}}\mathrm{h}_n^{(1)}(\mathrm{k}_j\mathrm{r}')][h_n^{(1)}(k_i r) + \widetilde{R_{\mathrm{i,i+1}}}j_n(\mathrm{k}_i\mathrm{r})]\cdot\widetilde{T_{ji}}\widetilde{M_i}\widetilde{M_j} \quad (5)$$

When $r\in$ region i （$i<j$）

$$F_n(\mathrm{r},\mathrm{r}') = [j_n(k_i\,\mathrm{r}) + \widetilde{R_{\mathrm{i,i-1}}}\mathrm{h}_n^{(1)}(\mathrm{k}_i\mathrm{r})][h_n^{(1)}(k_j r') + \widetilde{R_{\mathrm{j,j+1}}}j_n(\mathrm{k}_j\mathrm{r}')]\cdot\widetilde{T_{ji}}\widetilde{M_i}\widetilde{M_j} \quad (6)$$

Among them:

$$\widetilde{M}_{\mathrm{i}} = \frac{1}{1-\widetilde{R_{\mathrm{i,i-1}}}\widetilde{R_{\mathrm{i,i+1}}}} \quad (7)$$

$$\widetilde{T_{\mathrm{j,i}}} = \begin{cases} S_{\mathrm{j,j+1}}\cdots S_{\mathrm{i-2,i-1}}T_{\mathrm{i-1,i}} & i>j \\ S_{\mathrm{j,j-1}}\cdots S_{\mathrm{i+2,i+1}}T_{\mathrm{i+1,i}} & i<j \end{cases} \quad (8)$$

$$S_{i,i+1} = \frac{T_{i,i+1}}{1-R_{i+1,i}\widetilde{R_{i+1,i+2}}} \quad (9)$$

$$S_{i,i-1} = \frac{T_{i,i-1}}{1-R_{i-1,i}\widetilde{R_{i-1,i-2}}} \quad (10)$$

$$\widetilde{R_{i,i-1}} = R_{i,i-1} + \frac{T_{i-1,i}\widetilde{R_{i-1,i-2}}T_{i,i-1}}{1-R_{i-1,i}\widetilde{R_{i-1,i-2}}}$$

$$\widetilde{R_{i,i+1}} = R_{i,i+1} + \frac{T_{i+1,i}\widetilde{R_{i+1,i+2}}T_{i,i+1}}{1-R_{i+1,i}\widetilde{R_{i+1,i+2}}} \quad (11)$$

where R and T are obtained using the idea of generalized reflection coefficients.

RESULTS

In this section, previous results are compared to verify the accuracy of the equations and numerical results. We discuss the electromagnetic properties of the spherical “Earth-ionosphere” model, compare them with the results of the planar layered model, and analyze the physical mechanisms.

Comparison of the theoretical results

Peng (2013) gives numerical results for the electromagnetic field in the spherical waveguide of the Earth's ionosphere. The resistivity of the air is 10^{14} Ω • m, the resistivity of the ionosphere is 10^5 Ω • m, and the resistivity of the Earth is 1000 Ω • m. The height of the ionosphere from the ground is 85 km, the radius of the Earth is 6370 km, and the electric dipole moment is Idl = 1A • m.

These two results are basically consistent, as shown in Fig. 2. We further calculated the relative error between our results and Peng's results. The black dashed line indicates that the relative error is less than 1%.

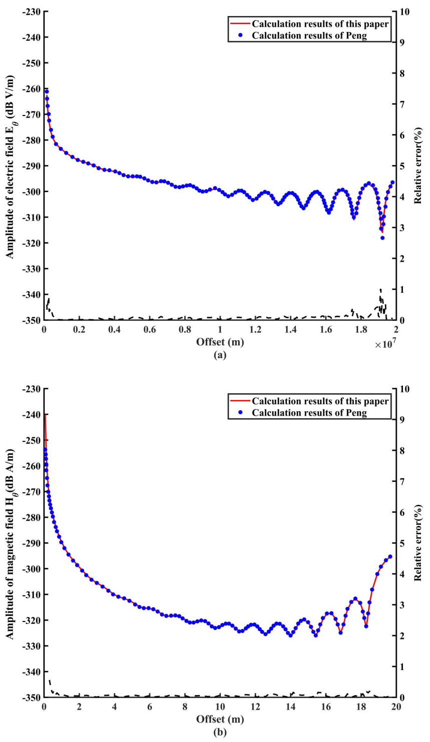

Figure 2 Comparison diagram of the electromagnetic field results. The red curve represents our result. The blue dot represents the results of Zhang. The black dotted line is the relative error. (a) Tangential electric field E_θ propagation curve. (b) Tangential magnetic field H_θ propagation curve

Comparison between planar and spherical model

In order to illustrate the effect of the curvature of the Earth on the electromagnetic field, we compare the calculations of a planar model (Li et al.,2015) with those of a spherical model based on the resistivity of the air and the ionosphere. The resistivities of the Earth are 10^{14} Ω • m, 10^5 Ω • m, and 1000 Ω • m, the height of the ionosphere from the ground is 100 km, the radius of the Earth is 6370 km, and the electric dipole moment is Idl = 250 A • 50 km. When the source-receiver distance is less than half of the radius of the Earth, the effect of the curvature of the Earth is very small, and the Earth's surface can be approximated to be a plane. Figure 3 shows that the two calculations agree well up to 3000 km, but the curves diverge as the distance increases.

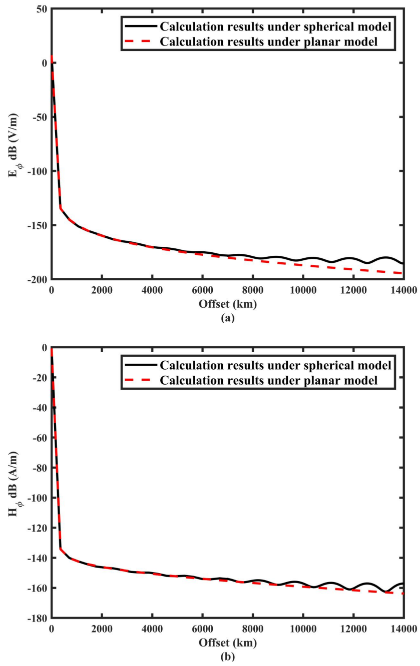

Figure 3. Comparison of electromagnetic fields at a frequency of 100 Hz under the spherical-planar model

It is found that the amplitude of the field oscillating in space may increase in spherical cavity models with larger offsets. In contrast to the conventional planar model, the field on the global scale does not decay with increasing source-receiver distance. The ionosphere, air, and Earth together form a horizontal infinite waveguide in the planar model; therefore, electromagnetic waves from a source can propagate in all directions. However, in the spherical model, the ionosphere, air, and Earth form a closed spherical waveguide. Electromagnetic waves trapped in the waveguide may resonate. In the near field, resonance is absent because the field emitted directly from the source is dominant and shows only geometric attenuation.

Conclusions

We have rederived the electromagnetic field expression under multilayer spherical media based on the idea of generalized reflection coefficients. The numerical results show that the difference between them and the horizontal model occurs at a transceiver distance of thousands of kilometers, and the effect of the curvature of the Earth should be taken into account.

References

HAN Qi, HU XiangYun, PENG Rong Hua. 2020. Spherical magnetotelluric modeling based on non-uniform source. Chinese Journal of Geophysics (in Chinese), 63(8): 3154-3166

Wait, J.R., Propagation of ELF electromagnetic waves and project Sanguine/Seafarer. 1977. 2(2):

p. 161-172.
Watson, G.N., Containing Papers of a Mathematical and P. Character, The diffraction of electric waves by the earth.1918. 95(666): p. 83-99.
Houdzoumis, V.A.J.R.S., Two modes of wave propagation manifested in vertical electric dipole radiation over a sphere.2000. 35(1): p. 19-29.
Barrick, D.E., Exact ULF/ELF dipole field strengths in the Earth-ionosphere cavity over the Schumann resonance region: Idealized boundaries. Radio Science, 1999. 34(1): p. 209-227.
Pan, W. and K. Li, Propagation of SLF/ELF electromagnetic waves. 2013: Springer Science & Business Media.
Wang, Y.-X., et al., Exact SLF/ELF underground HED field strengths in earth-ionosphere cavity and Schumann resonance. IEEE transactions on antennas, 2011. 59(8): p. 3031-3039.
Peng, H.Y.; Pan W.Y.; Guo L.X. Legendre function series method for ELF fields excited by horizontal electric dipole in earth-ionosphere cavity. Chin. J. Radio Sci. 2013, 28, 853–858. DOI: 10.13443/j.cjors.2013.05.006
Li, D.; Di, Q.; Wang, M.; Nobes, D. 'Earth–ionosphere' mode-controlled source electromagnetic method[J]. Geophysics. J. Int. 2015, 202, 1848–1858. DOI:10.1093/gji/ggv256.