

Sensitivity of Magnetotelluric Inter-station Transfer Functions

Gang WEN¹, Bo YANG¹, Xiaolei WU¹, Yixian Xu¹

¹School of Earth Sciences, Zhejiang University, Hangzhou, China, bo.yang@zju.edu.cn

SUMMARY

In two-dimensional (2D) and three-dimensional (3D) environments, magnetotelluric (MT) data is sensitive not only to the geoelectric structure below the site but also to any lateral anomalies around the measuring site. The lateral resolution of data inversion is related to the spacing of the site, while the depth resolution is related to the frequency. However, the existing theory does not provide a specific quantitative relationship between resolution and these variables. The sensitivity matrix provides a quantitative description of the impact of varying each model parameter on the data, thereby offering a valuable tool for analysing the resolution of data to the model. We developed a synthetic model to analyse the sensitivity of MT transfer function (impedance \underline{Z} and vertical magnetic transfer function \underline{T}) and horizontal magnetic interstation transfer function (\underline{M}) in different geoelectric structures. The results demonstrate that: (1) \underline{M} is sensitive to the conductor and almost unaffected by low resistance shielding effects; (2) \underline{M} has an approximate sensitivity amplitude to \underline{T} (much smaller than that of \underline{Z}), but the inclusion of the reference station makes the range of influence of \underline{M} on lateral geoelectric structures much larger than that of \underline{T} . In general, we believe that the resolution of anomalies beneath conductor and shallow geoelectric structures could probably be greatly improved if \underline{M} were added to the inversion.

Keywords: Magnetotelluric inter-station transfer function; Sensitivity; Model resolution.

INTRODUCTION

Except for remote reference (Gamble et al, 1979), traditional MT data processing and inversion methods assume that each station operates independently. Campanya et al (2016) conducted a synthetic 2D model test for the joint inversion of the impedance tensor \underline{Z} , the vertical magnetic transfer function \underline{T} , and the horizontal magnetic interstation transfer function \underline{M} under 3D conditions. They concluded that incorporating the interstation transfer function significantly enhances the recovery of anomalous bodies below and outside the profile and improves the resolution of the bottom and lateral boundaries of conductors.

Building on this, Kruglyakov and Kuvshinov (2019) developed a 3D inversion program for the impedance tensor and interstation transfer functions to test the 3D model. They argued that the joint inversion of \underline{Z} and other interstation transfer functions does not significantly improve the inversion results, and the single inversion of the horizontal electric interstation transfer function (\underline{H}) and \underline{M} is markedly

less effective than the inversion of \underline{Z} . However, there are some issues with the above simulations, such as only one model being tested and potential inadequacies in the model and observation system setup. Previous studies have used only one reference point in the model. Does the position of the reference point affect the resolution of the model?

The sensitivity matrix (\underline{J}) is the partial derivative matrix of the data vector \underline{d} with respect to the model vector \underline{m} . It is a crucial tool for assessing whether the observation system is sensitive to the preferred model. Rodi (1976) first applied the reciprocity theorem to calculate the sensitivity matrix of the two-dimensional MT response using the finite element method. Since then, this approach has been employed in numerous electromagnetic inversion algorithms (Kelbert et al, 2014).

To better understand the characteristics and physical significance of interstation transfer functions, we developed a 3D model to calculate the sensitivity matrix of \underline{Z} , \underline{T} , and \underline{M} using a modular system for electromagnetic inversion (ModEM, (Egbert and Kelbert, 2012; Kelbert et al, 2014)). We also ana-

lyzed the influence of different reference points on the sensitivity of $\underline{\mathbf{M}}$ and examined the sensitivity responses of $\underline{\mathbf{M}}$, $\underline{\mathbf{Z}}$, and $\underline{\mathbf{T}}$ in various geoelectrical structures.

EXPERIMENTAL DESIGN

The numerical experiment involved calculating the sensitivity matrix of a defined synthetic model with embedded 3D anomalies. These responses were used for a sensitivity test, which evaluated the effects of each anomaly on the different types of data: $\underline{\mathbf{Z}}$, $\underline{\mathbf{M}}$, and $\underline{\mathbf{T}}$. The sensitivity of stations located above a conductor (C1) and a resistor (R1) was evaluated, respectively.

We set the synthetic model with one cubic conductor (C1) having a resistivity of $1 \Omega \cdot m$ and one cubic resistor (R1) with a resistivity of $10^4 \Omega \cdot m$, each measuring 5.6×5.6 km, embedded in a $100 \Omega \cdot m$ half-space. The depth extension of these anomalies ranges from 0.16 to 12 km, separated by 7.2 km. We discretized the model core area with a lateral grid size of 0.8×0.8 km, and the layer thicknesses increase exponentially with depth, extending from 0 m to 49 km. The total number of grids in the synthetic model is $31 \times 31 \times 16$. The model includes 961 MT sites, with a distance of 0.8 km between them. The frequency range spans from 1000 Hz to 0.001 Hz, encompassing 24 frequency points.

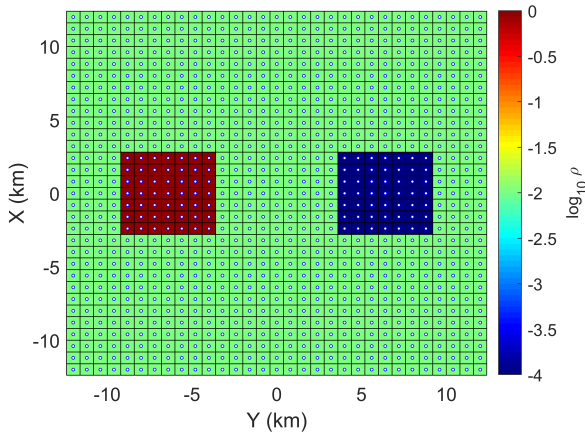


Figure 1: Map of 7 km depth slices for a combined R1 and C1 prismatic cylinder model. The resistivity of the dark red prismatic cylinder (C1) is $1 \Omega \cdot m$ and the dark blue prismatic cylinder (R1) is $10^4 \Omega \cdot m$. The white circles are MT sites, located in the center of the grid.

For the MT transfer functions, we computed the following three types: (1) the MT impedance tensor $\underline{\mathbf{Z}} = E^l/H^l$; (2) the vertical magnetic transfer function $\underline{\mathbf{T}} = H_z^l/H^l$; and (3) the interstation horizontal magnetic transfer function $\underline{\mathbf{M}} = H^l/H^r$. Here, H^l and H^r are two-component vectors comprising the horizontal magnetic components at the local site and the reference site, respectively. E^l is a two-component vector comprising the horizontal electric components at the local site, and H_z^l is the vertical component of the magnetic field recorded at the local site.

The sensitivity matrix $\underline{\mathbf{J}}$ is the partial derivative matrix of the data vector \mathbf{d} with respect to the model vector \mathbf{m} . It is a tool to assess whether the observation system is sensitive to the target. The basic formula is:

$$J_{ik} = \frac{\partial d_i}{\partial \sigma_k} \quad (1)$$

The change in conductivity of the k -th model block affects the data observed at the i -th observation site. Therefore, by examining the magnitude of $\underline{\mathbf{J}}$, it is possible to determine whether the observation system is sensitive to the target.

SENSITIVITY ANALYSIS

In this section, we discuss the sensitivity of three response types $\underline{\mathbf{Z}}$, $\underline{\mathbf{M}}$, and $\underline{\mathbf{T}}$ above the sites of R1 and C1. The sensitivity calculation module in ModEM (Egbert and Kelbert, 2012; Kelbert et al, 2014) is used to directly calculate the sensitivity matrix of the synthetic model. The sensitivity matrices of the actual parts of the components are then mapped onto the grid of the synthetic model. The value of each grid cell indicates the magnitude of the electromagnetic field change induced at the station by a change in its conductivity. To visualize the change in sensitivity, a logarithmic scale with a base of 10 is used to represent the sensitivity amplitude. Since the sensitivity of the model can change in both positive and negative directions, we use a color scale with warm and cool hues to represent the different modifications of the model, respectively.

Conductor

We first analyse the characteristics of the synthetic model, when the MT station is above the C1, under 4.5 Hz single-frequency time-harmonic vertically incident planar electromagnetic wave excitation. As shown in Figure 2, the inclusion of a reference station results in \underline{M} exhibiting a broader range of sensitivity to the surrounding structure than \underline{T} . The sensitivity amplitudes of \underline{M} and \underline{T} are comparable, although the difference in amplitude between \underline{M} and \underline{Z} is considerable. The maximum magnitude of \underline{M} is nearly equal to the minimum magnitude of \underline{Z} . Meanwhile, we also observe that the sensitivity amplitude and the range of influence of C1 are greatly reduced compared to those of the stations above R1 (Figure 3), due to the shielding effect of the conductor, which absorbs the electromagnetic wave energy as it propagates subsurface. Therefore, we believe that the inclusion of \underline{M} and \underline{T} in the inversion can improve the resolution of the conductor, and the inclusion of the reference station of \underline{M} will also improve the resolving power of the geoelectric structure around the reference station.

Resistor

As illustrated in Figure. 3, when the sensitivity stations are situated on R1, the sensitive areas of \underline{Z} and \underline{T} are considerably larger than those on C1. The off-diagonal elements of \underline{M} exhibit a comparable change in sensitivity to the diagonal elements of \underline{Z} , and the diagonal elements of \underline{M} exhibit a comparable change in sensitivity to the off-diagonal elements of \underline{Z} . The preceding characterization allows us to conduct a cursory analysis of the physical mechanism of \underline{M} . The magnetic field component H_l^x of the x-direction polarized vibration of the local station induces an electric field E_l^y around it, which in turn generates the y-direction vibration. The x-square magnetic field component H_r^x is induced at the reference station as a result of the x-direction polarized vibration of the local station. Consequently, the response of M_{xx} is characterised similarly to Z_{xy} due to a clear physical connection between them. The same can be said of the other components. M_{xx} and M_{yy} retain the capacity to discriminate against conductor, even when the reference station of \underline{M} is situated within a half-space.

However, it can be observed that the positive and negative modifications of the model are reversed when the locations of the reference stations are dif-

ferent. It remains to be seen whether this feature is determined by the resistivity value below the reference station or the relative position of the reference station to the local station, with further models required to verify this. The amplitude distribution of \underline{T} is considerably simpler than that of \underline{M} and \underline{Z} . The modeled modification of \underline{T} is concentrated mainly along the direction of the denominator H_l , and \underline{T} is similarly unresponsive to conductor. The sensitivity response characteristic of \underline{T} is relatively straightforward, exhibiting only a slight variation in the direction of the horizontal component.

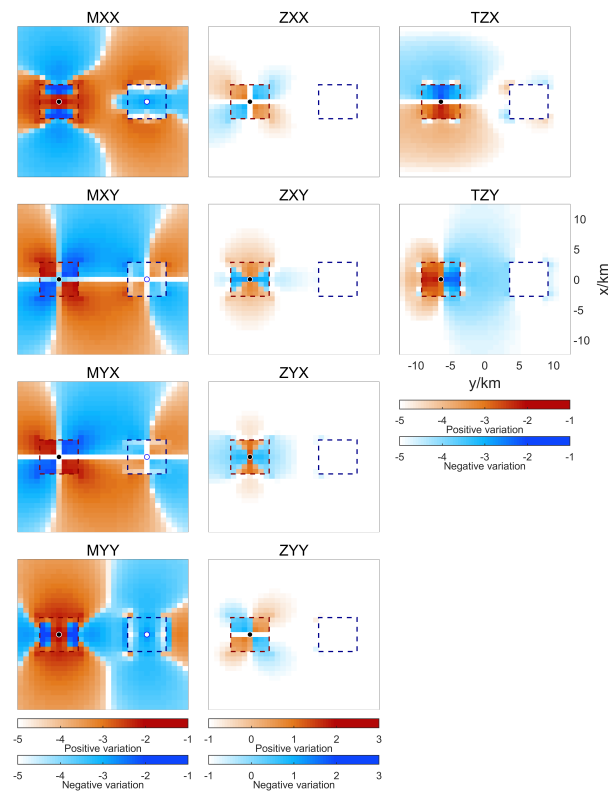


Figure 2: Depth slice (760 m) of the sensitivity amplitude response (\underline{M} , \underline{Z} , and \underline{T}) for C1 in the synthetic model. The frequency of MT plane wave field source is 4.5 Hz. The red and blue dashed lines delineate the boundaries of C1 and L2 in the model, the black point represents the sensitivity analysis site, and the white point represents the reference site.

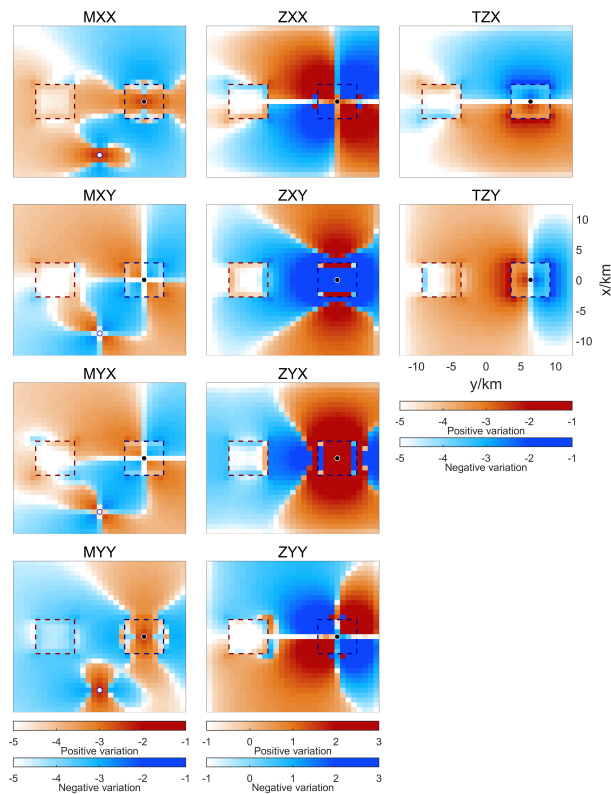


Figure 3: Depth slice (760 m) of the sensitivity amplitude response (\underline{M} , \underline{Z} , and \underline{T}) for R1 in the synthetic model. The frequency of MT plane wave field source is 4.5 Hz. The red and blue dashed lines delineate the boundaries of L1 and L2 in the model, the black point represents the sensitivity analysis site, and the white point represents the reference site.

CONCLUSIONS

We quantified the sensitivity response characteristics of the three transformation functions (\underline{Z} , \underline{M} , and \underline{T}) in a synthetic model through the sensitivity calculation module of ModEM. The results of our analysis indicate that \underline{M} attenuates the shielding effect of electromagnetic waves in conductor, rendering it more sensitive than \underline{Z} and \underline{T} in conductor. The response exhibited by M_{xx} is analogous to that observed in Z_{xy} , due to the clear physical connection of them (M_{yy} is the same as M_{xx}). The sensitivity

response characteristic of \underline{T} exhibited only a slight variation in the direction of the horizontal component. Consequently, it is postulated that the addition of \underline{Z} , \underline{M} , and \underline{T} to the inversion will result in a notable enhancement of the model resolution during the inversion.

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