Hierarchical transdimensional Bayesian inversion of 2D magnetotelluric data

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SUMMARY

In this study, we present an algorithm for 2D inversion of magnetotelluric (MT) data. The algorithm is based on the transdimensional Markov chain Monte Carlo scheme for estimating the probabilistic conductivity model for the MT data. The concept of the Voronoi cell is used to parameterize the earth. The number of Voronoi cells used to represent the model is treated as unknown. Additionally, the noise level is also treated as unknown and is estimated thus making the approach completely data-driven. As millions of forward calls are required to create an ensemble that is a good approximation of the posterior probability density, we employ a multi-resolution finite difference scheme to speed up the computations. In this scheme, the mesh used for computing forward responses is represented by vertically stacked subgrids. The horizontal grid resolution of subgrids decreases with depth. The algorithm is tested over synthetic data. The results demonstrate the estimation of noise levels and uncertainty quantification in the model parameters.

Keywords: Magnetotelluric; Transdimensional Bayesian Inversion; Multiresolution.

INTRODUCTION

Magnetotelluric (MT) methods play a vital role in estimating subsurface electrical properties (Newman et al, 2008). In this method, the information about sub-surface electrical conductivity distribution is obtained through the inversion of recorded datasets. The traditional gradient-based approaches primarily focus on minimizing an objective function comprising data misfit and model regularization terms (Constable et al, 1987). However, these methods often lack robustness in providing insights into model uncertainty due to their reliance on a single model. Given the inherent nonuniqueness of the MT inverse problem, assessing uncertainty in model parameters becomes crucial. As computational capabilities have advanced, more sophisticated algorithms have emerged to address these challenges. In this study, we employ a sampling-based method called Transdimensional (trans-D) Bayesian inversion (Green, 1995). This method draws samples from uniform prior distributions, generating an ensemble of candidate models that fit the data, thus allowing for the extraction of statistical information about model parameters (Mosegaard and Tarantola, 1995). Furthermore, by implementing a reversible jump Markov chain Monte Carlo (rj-McMC) algorithm, we adaptively adjust model complexity to a level compatible with the data (Green, 2003). To speed up the computation of forward responses, we employ a multi-resolution (MR) approach (Cherevatova et al, 2018). This approach involves partitioning the modeling domain into a vertical stack of several subgrids. Finer grids are utilized in shallower regions to capture near-surface resistivity changes, while coarser grids suffice for deeper areas where electromagnetic wave behavior is diffusive. The synthetic results present various statistical measurements (such as the mean, median, mode, and standard deviation) that facilitate the precise retrieval of the true model's characteristics.

BAYESIAN INVERSION FRAMEWORK

The trans-D Bayesian inversion relies on Bayes's theorem, where the posterior probability distribution function in the Bayesian inversion-based framework is linked with the likelihood function and prior as:

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m})p(\mathbf{m}), \tag{1}$$

Here, **m** represents the vector of inversion model parameters, **d** is the observed data vector, *p*(**m**|**d**) is the posterior, which denotes the probability of the model given the observed data, *p*(**d**|**m**) is the likelihood, representing the probability of the data given a model, and *p*(**m**) is the prior. The concept of Voronoi cells is employed to parameterize 2-D model space, where each Voronoi cell is defined by three parameters: two for coordinates and one for resistivity value. Additionally, an unknown noise parameter $\boldsymbol{\lambda}$ is defined to relate the posterior noise $\boldsymbol{\sigma}^{\boldsymbol{post}}$ to the prior noise $\boldsymbol{\sigma}^{\boldsymbol{prior}}$ as:

$$\boldsymbol{\sigma}^{\boldsymbol{post}} = \boldsymbol{\lambda}\boldsymbol{\sigma}^{\boldsymbol{prior}} \tag{2}$$

The model parameter vector **m** comprises four parameters:

$$\mathbf{m} = [n, \mathbf{c}, \boldsymbol{\rho}, \boldsymbol{\lambda}] \tag{3}$$

where n represents the number of Voronoi nuclei, **c** denotes the coordinates of these nuclei, $\boldsymbol{\rho}$ represents the logarithmic resistivity values corresponding to each node, and $\boldsymbol{\lambda}$ represents the noise hyperparameter. The generation of a new model involves random perturbation in the current model, which can include adding a new cell (birth step), deleting a cell (death step), changing resistivity, altering the position of a cell, and adjusting the noise parameter. The acceptance or rejection of the newly proposed model is based on an acceptance ratio given by the Metropolis-Hastings-Green criterion. Accepted models are added to the ensemble, and the process continues until a sufficient number of models are collected. To expedite convergence, a delayed-rejection scheme, and parallel tempering are also incorporated in the algorithm (Sambridge, 2014).

We employ the finite difference method to discretize the second-order partial differential equation at each node of the grid. The 2-D domain is divided into a vertical stack of several subgrids, with finer grids used to capture near-surface small resistivity changes and coarser grids preferred for deeper parts. A schematic representation of a 2-D MR grid, consisting of four subgrids, is shown in Figure 1.

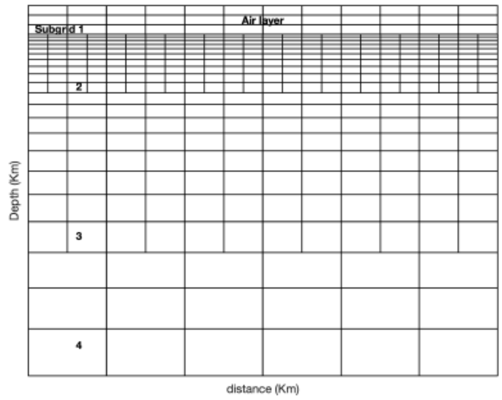

Figure 1: A schematic representation of multi-resolution 2-D grid consisting of a vertical stack of four subgrids

Results and Discussion

The numerical study was performed on a workstation equipped with two 2.3 GHz 24-core Intel Xeon processors and 192 GB of RAM. A synthetic model consists of two layers, as shown in Figure 2. The first layer is 22.5 km thick and has a resistivity of 100 Ω-m, while the second layer, with a resistivity of 10 Ω-m, extends infinitely. Within the first layer, two adjacent blocks measuring 22.5 km $\times$ 22.5 km have resistivities of 10 and 1000 Ω-m, respectively. The upper surface of these blocks aligns with the air-earth interface, marking the model's center. The model is discretized into a grid of 60 $\times$ 38 cells, each with horizontal dimensions of 2.5 km. Vertical discretization begins with a grid mesh size of 100 m below the surface, and the thickness of subsequent layers increases by a factor of 1.2. Additionally, 40 cells are padded around the central region, with 10 in each horizontal direction, progressively widening. Impedances were generated for 13 logarithmically equidistant periods ranging from 0.1 to 1000 s for both TE and TM modes. Twenty stations are evenly spaced along a 100 km profile, with a 5 km inter-station spacing. To mimic the real field scenario, a 2%, and 3% Gaussian noise was added to TE and TM mode impedances. The data errors were set to 1% of impedance to see the estimation of the noise hyperparameter.

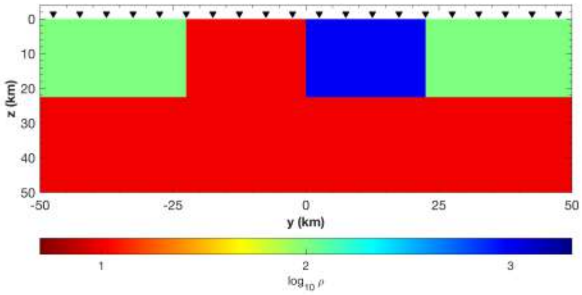

Figure 2: Synthetic resistivity model. The black triangles at z = 0 m denote MT site locations.

For the inversion run, the number of Voronoi cells can vary from 3 to 150, and these cells are allowed to be located between [-60, 60] and [0, 100] km along the y- and z-directions, respectively. Bounded uniform distributions on the range of [-1, 6] were set for the log10(resistivity) of Voronoi cells. A total of 15 Markov chains were run in parallel for 6×10^5 iterations. Out of these 15 chains, 10 chains are at the target temperature of 1, while the others are at temperatures logarithmically spaced between 1 and 10. It is observed that the chains converge rapidly to the high posterior probability regions within a few thousand iterations. Based on visual inspection of the convergence plot, the first 1×10^5 models were discarded from each chain as part of the burn-in process. From the remaining portion of the chain, every 100th model was selected to form the final ensemble, as adjacent models in a chain can be correlated. The final solution to the inverse problem consists of 50,000 models, where the model parameters are distributed according to their posterior marginal PDF.

While individual models in the ensemble may appear discontinuous and unrealistic, statistical measures such as mean, median, mode, standard deviation (SD), and others are employed to provide a smoother representation of the subsurface resistivity distributions. As demonstrated in the mean, median, and mode maps (Figures 4 (a-c)), both layers are well-resolved, and the resistive and conductive blocks in the first layer are accurately represented in terms of their position and associated resistivity values. The minimum and maximum resistivity values in the mean map are 9.2 and 1004 Ω-m, respectively. The SD map in Figure 4d illustrates that the major portions of the resistivity model are well-constrained. The highest SD is observed along the interfaces between the resistive and conductive blocks, as well as between the resistive block and the second layer. These zones, known as Uncertainty loops (Galetti et al, 2015) are due to the uncertainty in the position of the interface. Figure 3 shows the posterior probability density function (PDF) for the noise hyper-parameter $\boldsymbol{\lambda}$ for both the TE and TM modes. The peaks occur at 2.3 and 3.4 for the TE and TM modes, respectively, thus demonstrating the ability of the algorithm to estimate the data error levels.

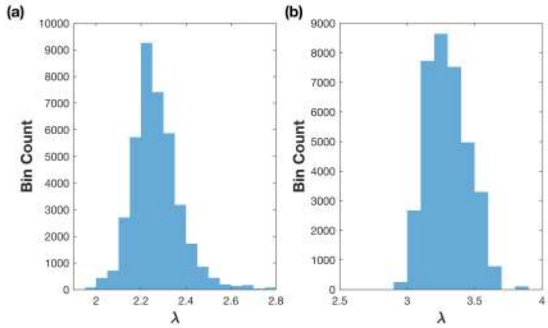

Figure 3: PDF on noise parameter $\boldsymbol{\lambda}$ for (a) TE mode (b) TM mode. PDF = Probability Density Function

Conclusions

This study successfully demonstrates the inversion of 2-D MT synthetic data with a hierarchical trans-D Bayesian approach. The posterior on the noise hyper-parameter is estimated for both the TE and TM modes. Incorporating MR grids with finite difference discretization of the EM field improves the efficiency of forward modeling, consequently speeding up the inversion process. The algorithm provides insight into model uncertainty by generating an ensemble of models, a feature lacking in traditional gradient-based inversion methods, which typically produce only a single model.

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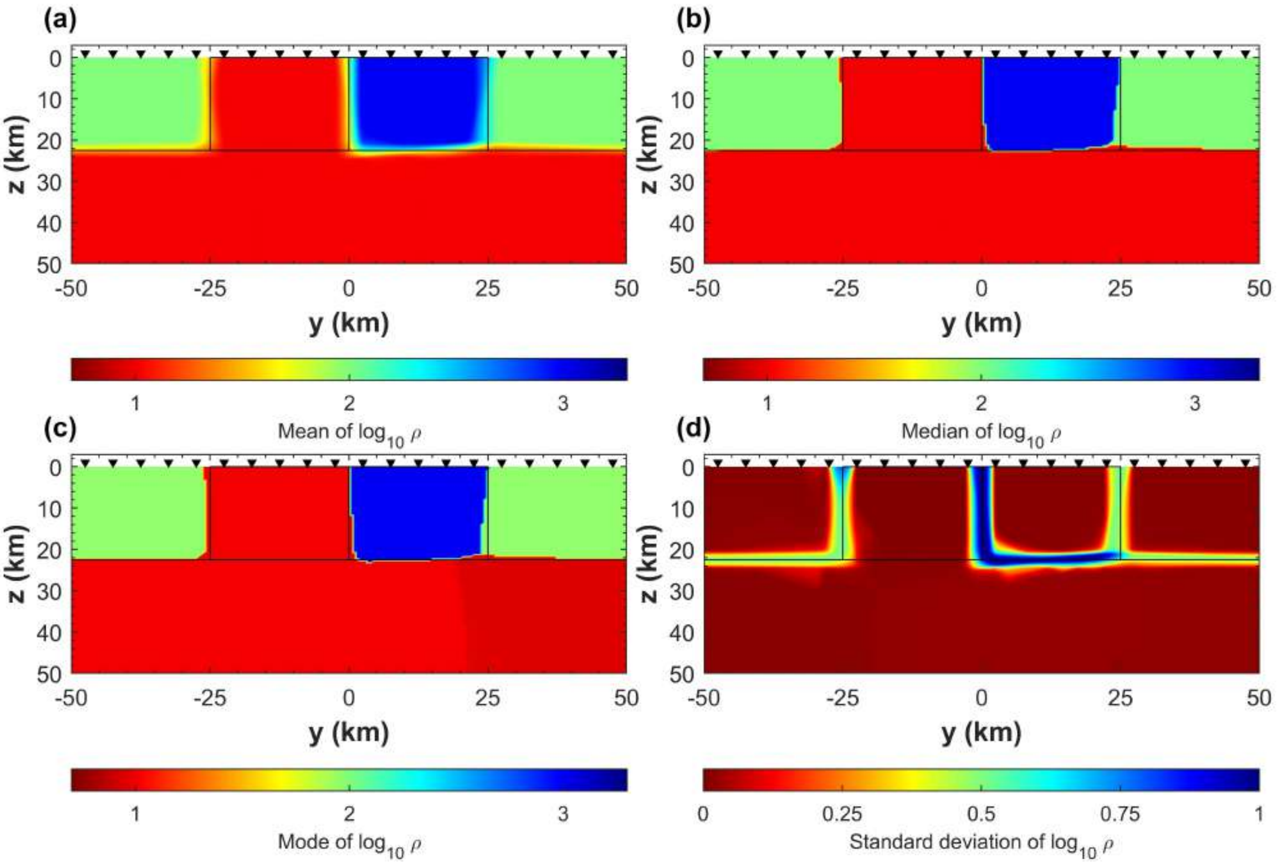

Figure 4: Inversion results for the synthetic model in Figure 2 using the algorithm (a) Mean, (b) Median (c) Mode (maximum posteriori), (d) Standard deviation. The black solid triangle at the air-earth interface represents the MT stations.