

An upscaling study for electromagnetic methods with infrastructure

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SUMMARY

Steel infrastructure, and in particular steel-cased wells, are often integral elements that need to be considered in geoscience problems. Yet their presence generates challenges for the simulation and inversion of electromagnetic data. Recently, there have been advancements in numerical strategies for approximating casings using highly refined tetrahedral or OcTree meshes, as well as the development of strategies that define steel casings on the edges of meshes in finite element or finite volume simulations. There are still questions about how to choose physical properties that approximate the casing in these approaches. In this abstract, we define an inverse problem to estimate the electrical conductivity and magnetic permeability of a solid cylinder that approximates a hollow steel-cased well. These values can then be used in a simulation that approximates the casing on a coarse mesh or on the edges of a mesh. By performing simulations and inversions at a range of frequencies, we also discuss physical insights that can be gained, such as seeing when inductive effects begin to dominate the recovered physical property values.

Keywords: Physical properties, Magnetic Permeability, Inversion, Borehole methods

INTRODUCTION

Electromagnetic (EM) methods are increasingly being applied in settings with steel infrastructure, including steel-case wells. For example in applications including carbon capture and storage, geothermal, wastewater injection, and casing integrity assessment steel cased wells often need to be included in simulations and inversions of EM data. Although steel casings present a challenge for numerical modelling, because of their geometry and extreme physical properties, they can be advantageous for helping detect targets at depth as they can serve as “extended electrodes” in grounded source DC resistivity and EM experiments (Schenkel and Morrison, 1990; Kaufman, 1990).

Strategies for handling infrastructure in numerical simulations have advanced in recent years and there are now multiple approaches available. Tetrahedral or OcTree meshes that are highly refined around infrastructure can be used (Um et al, 2015; Haber et al, 2016) with the computational cost depending upon the level of refinement. Cylindrical discretizations allow the details of the casing to be captured in a mesh without undue computational cost, but they are restricted to simulating vertical wells (Heagy and Oldenburg, 2019). Some authors have adopted strategies for replacing the casing

with an “equivalent source”, but these methods have the drawback that the source term needs to be re-computed for any changes in the physical property model because EM is nonlinear in electrical conductivity (Cuevas, 2013).

More recently, methods which modify the discretization of Maxwell’s equations to allow conductive structures to be discretized on the edges or faces of the mesh, as well as cell centers, have recently been developed Weiss (2017). This was first developed for DC resistivity and has since been extended to EM (Hu et al, 2022; Cowan et al, 2024). These approaches have the advantage of being able to perform simulations with thin, highly conductive structures without substantially increasing the computational cost. Additionally, by working with physical properties, one avoids the need to recompute an approximate source term when making model updates in an inversion. However, it isn’t clear how to handle magnetic permeability in these approaches.

Whether using standard codes or discretizing conductive features on the edges of a mesh, we face the question of how to assign physical properties on a coarser mesh. For grounded sources in the DC limit, it is well established that choosing a conductivity that preserves the product of the conductivity

and the cross-sectional area of the casing is the best choice (Kaufman, 1990). However, as frequency increases, we know that eventually skin-depth effects will dominate and this approximation no longer holds. Therefore, there must be a transition between these two end-member scenarios for some frequency range. Furthermore, steel has a substantial magnetic permeability ($\mu \sim 50 - 150\mu_0$) and we showed in (Heagy and Oldenburg, 2023) that this can impact data, even at relatively low frequencies.

This abstract investigates the question of how to discretize infrastructure by posing an inverse problem to assign physical properties to a solid cylinder that approximates the casing. This follows the idea of “upscaling” in electromagnetics (Caudillo-Mata et al, 2017; Schwarzbach and Haber, 2018).

SETUP AND MOTIVATION

The geometry we consider is shown in Figure 1a. A 500m long steel-cased well with a conductivity of 5×10^6 S/m is in a $10\Omega\text{m}$ background. The well has a diameter of 10cm and a 1cm thickness. We excite a response with a 1A current dipole that is 10m long and positioned 10m beneath the casing. This is a cylindrically symmetric problem and we use the cylindrical mesh implementation in SimPEG for all of our computations (Heagy and Oldenburg, 2019). The mesh has a radial discretization of 2.5mm cells in the region of the casing and a vertical discretization of 1m. The question we aim to address is: if we replace the hollow cased well (Fig. 1a) with a solid cylinder (Fig. 1b) what physical properties should we assign this cylinder?

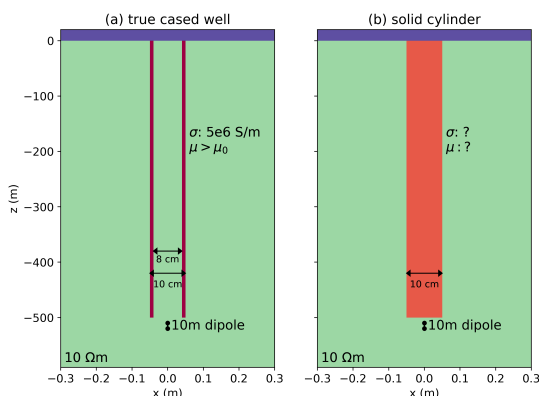


Figure 1: Setup of (a) the true, hollow, steel-cased well, and (b) solid cylinder.

To help motivate our study, we simulate the electric field data that would be measured at the surface for the true hollow-cased well excited by a current source at 2Hz and 10Hz. We examine the real part

of the radial component. These simulations are performed considering a range of permeabilities, and the results are shown in Figure 2(a,b). Next, we run simulations where we replace the hollow cased well with a solid cylinder that has a conductivity that preserves the product of the conductivity and cross-sectional area (e.g. the solution at DC), for this example, that is a conductivity of 1.8×10^6 S/m. The magnetic permeability of the cylinder is assumed to be the free-space permeability μ_0 . In Figure 2 (c) and (d) we show the difference between the true data and those simulated with the solid cylinder; Figures 2 (e) and (f) show that difference as a percentage. At low frequencies and low permeabilities, the DC approximation is suitable and the two solutions are in good agreement. For the 2 Hz source, only at the largest permeabilities do we see a difference of 1-3%. However, in the 10 Hz data, we see that if the permeability of the well exceeds $100\mu_0$ (which is feasible for steel), then the difference exceeds 10%. For a well with a permeability of $150\mu_0$, the difference is 20-30%.

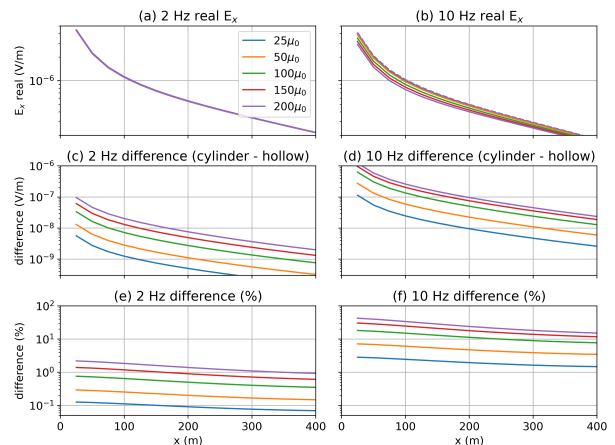


Figure 2: Simulated real, radial electric field at the surface at (a) 2Hz and (b) 10 Hz; (c, d) difference between simulated data for hollow cased well and a solid cylinder; (e, f) difference as a percentage.

METHODS

We pose the upscaling problem as an inverse problem of the form

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \|\mathbf{W}_d(\mathbf{F}[\mathbf{m}] - \mathbf{d}^{\text{true}})\|^2 \quad (1)$$

where the model, \mathbf{m} contains 2 parameters, the log-conductivity of the solid cylinder and the relative permeability ($\mu_r = \mu/\mu_0$) of the cylinder. \mathbf{F} is the forward modelling operator, \mathbf{d}^{true} are the data associated with the true, hollow-cased well, and \mathbf{W}_d is a diagonal weighting matrix that contains the inverse of the standard deviation of each datum. We do not

include a regularization because there are only two model parameters that we invert for.

We have flexibility in how we define the “data” to be fit in the inversion. In this case, we are interested in having the recovered model reproduce measurements that would be made at the surface as well as creating the same (or similar) response in the subsurface. For example, if the end goal is a monitoring experiment, we would want to be able to produce the same excitation (e.g. amplitude and geometry) of the fields in the subsurface. So for this example, we choose our data to be the radial and vertical electric fields on a 25m × 25m grid that extends 500m horizontally and 600m vertically. We use both the real and imaginary components of the field.

No noise is added to the data and we use small values to define the data uncertainties, namely a 0.5% relative error and a 10⁻⁸ V/m floor. We allow the inversion to run beyond the target misfit and stop it when minimal progress is being made. The optimization is performed using Inexact Gauss-Newton and a lower bound of 1 is imposed for the relative permeability.

CONDUCTIVE WELL

We begin with a one-parameter problem where the goal is to find the upscaled conductivity of a non-permeable well. We vary the conductivity of the casing from 5 × 10⁵ S/m to 10⁷ S/m and fix the magnetic permeability to be μ₀. We run simulations and inversions for 36 frequencies over 5 decades from 0.1 Hz to 10⁴ Hz. In Figure 3 (a) we show the resulting unweighted RMS misfit between the predicted and true data, and Figure 3 (b) shows the recovered conductivities.

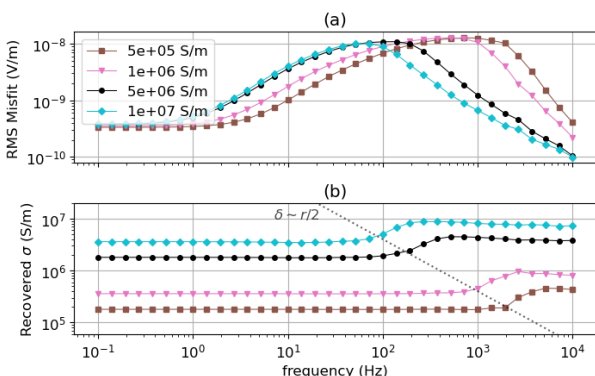


Figure 3: (a) RMS misfit, and (b) recovered conductivity.

For low frequencies, we can see that the best-fitting conductivity agrees with the DC approximation that preserves the product of the conductivity and the

cross-sectional area of the casing. As frequency increases, we see that a transition starts approximately when the skin depth is approximately half of the radius of the casing ($r/2$), and the recovered conductivity plateaus to a value near the true casing conductivity.

CONDUCTIVE, PERMEABLE WELL

Next, we set the conductivity of the casing to 5 × 10⁶ S/m and run simulations and inversions varying its magnetic permeability from μ₀ to 200μ₀. We allow the inversion to estimate both the conductivity and the permeability of the cylinder approximating the casing. Figure 4 shows the results including (a) the RMS misfit, (b) the recovered conductivity (S/m) and (c) the recovered relative magnetic permeability.

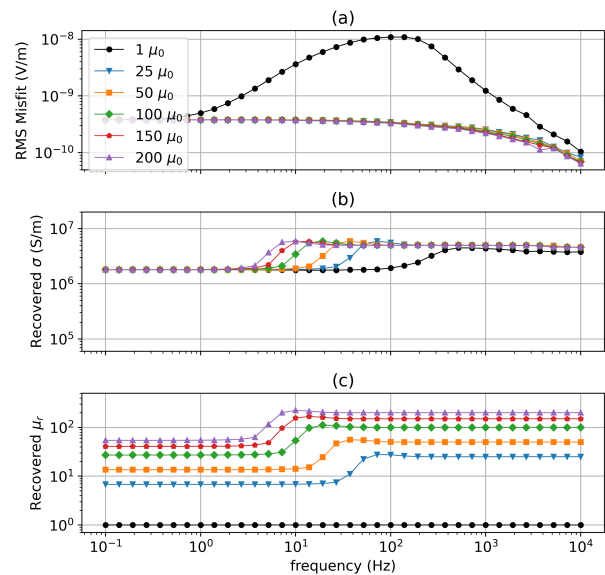


Figure 4: (a) RMS misfit, (b) recovered conductivity, and (c) recovered permeability.

Again, we see that at low frequencies, the DC approximation for the conductivity is appropriate. For the magnetic permeability, shown in Figure 4, we see that it also plateaus to a constant value for low frequencies. As the frequency increases, we again see the influence of inductive effects and at sufficiently high frequencies the best-fitting conductivity and permeability plateau at the true values. The low-frequency approximation appears to be linear in the true permeability, thus we expect that there is a rule-of-thumb that could be developed based on the geometry and permeability of the casing for defining an upscaled value. Further investigation into different geometries will be needed to assess the parameters controlling this value.

DISCUSSION & CONCLUSIONS

In this abstract, we have used inversion to estimate physical properties that can represent steel infrastructure on a coarse scale. This approach can be used to estimate physical properties to be input into a 3D code, for example using a standard OcTree or Tetrahedral discretization that approximates the casing on a coarse scale. Alternatively, the estimates could be used in codes that approximate electrical conductivity on the edges of a mesh, however, these approaches do not yet account for magnetic permeability. An advantage of working with physical properties rather than an approximate source term is that the estimated physical properties are less impacted by the host formation than an approximate source term, which depends non-linearly on the physical properties of the formation. Furthermore, we expect the estimated physical properties to be robust to the geometry of the casing (e.g. for deviated or horizontal wells), whereas it is less obvious if an approximate source term would be.

In our examples, we see that there is a transition from the best-fitting values obtained at low frequencies to those obtained at “higher” frequencies where we see the impacts of inductive effects. For the examples we illustrated, this transition could occur at frequencies as low as 5 Hz. This exact number will depend on multiple properties including the geometry and properties of the casing. At low frequencies, the best-fitting conductivity is the one that preserves the product of the conductivity and cross-sectional area. At higher frequencies, the best-fitting conductivity is approximately equal to the true conductivity of the casing. Magnetic permeability follows a similar trend. At low frequencies, the recovered permeabilities depend linearly upon the true permeability, and investigating if there is a heuristic that can be followed, similar to the DC approximation for conductivity, is an avenue of future research. Transitioning to higher frequencies, the recovered permeability value equals the true permeability.

These results have several important implications. First, if we are interested in assessing the true casing properties in a field experiment, running a high-frequency experiment will be directly sensitive to the true casing properties. Thus, we expect that it should be straightforward to estimate those values from high-frequency data. Second, if we consider the time domain, these results imply that to accurately simulate the responses, time-varying values of conductivity and magnetic permeability should be considered. However, it may be that for specific survey designs and time ranges of interest, an approxi-

mate scalar value can be appropriately chosen. This will be an avenue for future investigation.

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