A time domain finite element iterative solution scheme for transient electromagnetic method

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SUMMARY

This paper proposes a new scheme for calculating the transient electromagnetic response in the time domain. It achieves efficient computation for large-scale underground models through the iterative solution of large sparse equation systems. A set of appropriate time steps and spatial grid discretization parameters are provided for commonly used step waveforms, enabling efficient computation for different models while meeting the requirements of computational error. Overall, this computational scheme has relatively low requirements for time steps and spatial grids, making it suitable for the wide and complex application scenarios of transient electromagnetic methods.

Keywords: Transient electromagnetic method; Finite element method

INTRODUCTION

Transient electromagnetic (TEM) method is an important electromagnetic exploration technique that observes the secondary field generated by underground anomalies through a single excitation of the field source to obtain resistivity information about the underground media. Due to its low cost, high efficiency, strong anti-interference capability, and ease of construction, the TEM method is currently widely used in fields such as mineral exploration, groundwater exploration, mine tunnel advanced detection, and marine mineral surveys.

The three-dimensional forward modeling of the TEM method is mainly divided into two types of calculation methods: frequency domain and time domain. The frequency domain method utilizes the Maxwell's equations in the frequency domain to calculate the frequency domain response and then converts it to the time domain through fourier transform based methods(FT). The time domain method involves discretizing the Maxwell's equations in the time domain and solving them directly through iterative techniques. Currently, there are relatively few efficient TEM algorithms for large-scale underground models. Only a few algorithms based on Krylov methods and parallel solution algorithms using GPUs have been developed, which have high requirements for computing equipment. This paper proposes a time-domain calculation scheme based on iterative solution, which effectively reduces the memory required for large-scale numerical calculations. Meanwhileï¼MPI parallelism can be utilized to improve computational efficiency.

METHODS

The Maxwell's equations in the time domain (Faraday's law of electromagnetic induction and Ampere's law) can be expressed in the following form (Wang and Hohmann, 1993)ï¼

$$\nabla \times E = -\frac{\partial B}{\partial t}. \tag{1}$$

$$\nabla \times \frac{1}{\mu} E = J + \sigma \frac{\partial E}{\partial t}. \tag{2}$$

In the formula, E is the electric field intensity, B is the magnetic flux density, J is the current density, μ is the magnetic permeability, and σ is the electrical conductivity. For transient electromagnetic methods, under quasi-static conditions, the displacement current term can be neglected (Yin and Hodges, 2005). By combining the above two equations, we have:

$$\frac{1}{\mu}\nabla\times\nabla\times E\left(r,t\right)+\sigma\frac{\partial E\left(r,t\right)}{\partial t}+\frac{\partial J_s\left(r,t\right)}{\partial t}=0. \quad (3)$$

This is the double curl equation of the electric field in the time domain. To solve this equation in the time domain, we employ the finite element method for spatial discretization and the backward Euler method for temporal discretization, resulting in the following governing equation (Um et al, 2010):

$$\left(3M+2\Delta tS\right)E^{i+2}\left(t\right)=M\left(4E^{i+1}\left(t\right)-E^{i}\left(t\right)\right)-2\Delta tJ^{i+2}. \quad (4)$$

In the formula, M is the mass matrix, S is the stiffness matrix, E is the discretized electric field, J is the field source, and i is the time step.

By loading the electric field E onto the edges, we can transform the above equation into a large sparse system of equations. To solve this system, we use the generalized minimum residual method (FGMRES) (Liu et al, 2022). Compared to direct solution methods, iterative solution methods require independent computations for each time step, but they have a smaller memory footprint. To further improve the computational efficiency of the iterative solution, we use a domain decomposition algorithm for parallel computation. For the calculation of transient electromagnetic responses for different models, we only need to provide a reasonable time step template and similar spatial model parameters, and we can obtain stable results with strong applicability. For the required time channels, they can be obtained through interpolation with minimal loss of accuracy.

RESULT

Layered Model

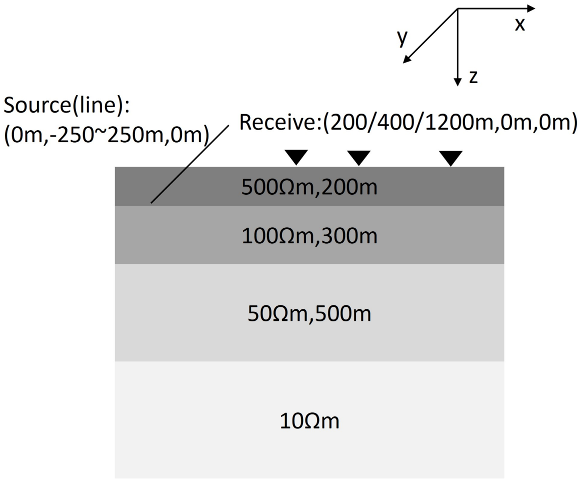

Figure 1: Sketch of the layered model.

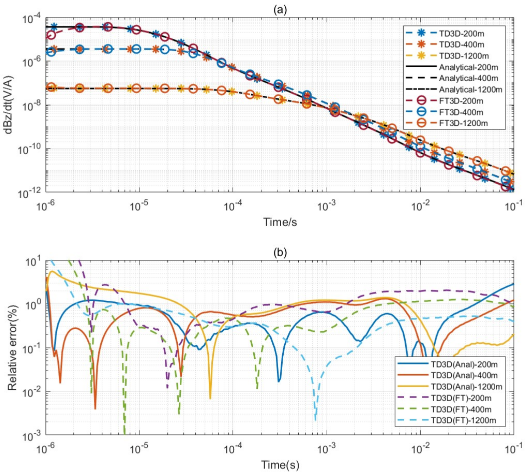

Figure 2: Responses for the Layered model. We present the (dBz/dt) responses at three receivers(a). Relative errors of (dBz/dt) compared with 1D analytical and FT result (b).

We calculated the responses for a layered model, as depicted in Figure 1 The calculation region is divided into four layers, with 426,086 cells. A total of 310 time steps were used, and the calculation was performed in parallel on 6 cores, taking approximately 10 minutes with a maximum memory usage of 3.3GB. We compared the results of our calculation with those obtained using a one-dimensional semi-analytical solution and a three-dimensional frequency domain method. As can be seen, the time-domain calculation results align well with the results from the other two algorithms, demonstrating the correctness of our algorithm (Figure2).

Complex Conductor at a Vertical Contact

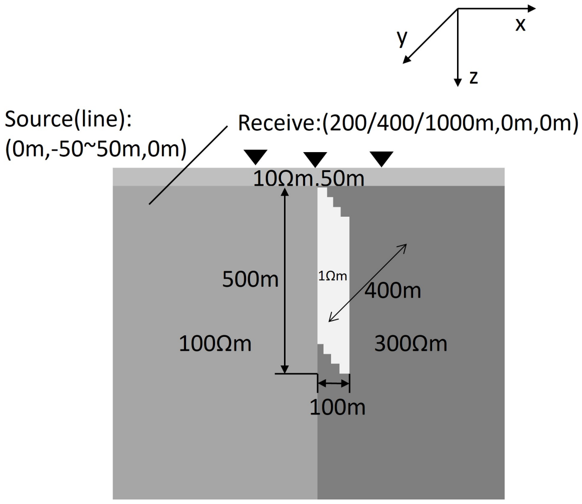

Figure 3: Sketch of the complex model.

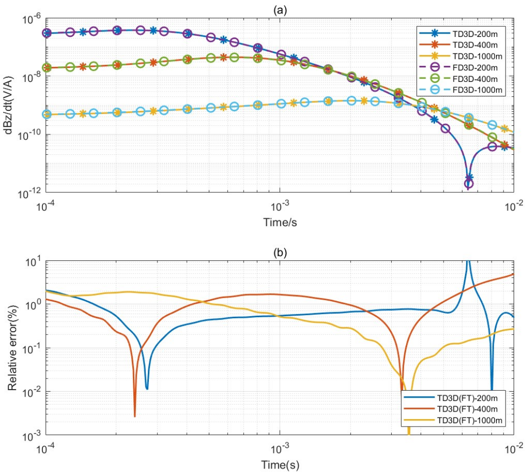

Figure 4: Responses for the complex model. We present the (dBz/dt) responses at three receivers(a). Relative errors of (dBz/dt) compared with FT result (b).

We performed calculations for a complex conductor at a vertical contact, as shown in Figure 3 The model has 391791 cells, and the time step is the same as mentioned earlier. The calculation was performed in parallel on 6 cores, taking approximately 9 minutes, with a maximum memory usage of 3GB. We compared the results of our calculation with those obtained using a three-dimensional frequency domain method and observed a good agreement between the two algorithms (Figure4).

CONCLUSIONS

This article introduces a novel TEM forward modeling solution. By using the FGMRES iterative solution algorithm, we avoid the decomposition of the large sparse system of equations on the left side, which will cause significant memory usage. The advantage of iterative solution lies in its flexibility in selecting the time step and its lower memory consumption, especially when dealing with complex waveforms and large-scale modeling environments. We verified the correctness of the algorithm by simulating the TEM response in a uniform half-space. Furthermore, we performed forward modeling for various models based on tetrahedral meshes, and the algorithm produced stable response results, demonstrating strong applicability. Finally, we conducted simulations for complex underground environments. The results showed that our algorithm consumes less memory when computing larger models, making it suitable for large-scale TEM forward modeling.

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