A potential function approach for modelling electromagnetic induction responses over Earth models with two-dimensional boundaries

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SUMMARY

Three-dimensional natural-source electromagnetic induction modelling typically uses one-dimensional background models for studying localized conductivity structures. In magnetotelluric data modelling, there are scenarios where exact two-dimensional boundary surfaces need to be honoured such as studying of terrain effects on the magnetotelluric data. Our focus in this study is about coastal and topographic (bathymetry) effects on marine magnetotelluric data where two-dimensional, exact boundaries are important. Due to the large scale characteristic of marine models, the standard Helmholtz equation for the electric field, which is frequently used for three-dimensional forward modelling of magnetotelluric data, becomes less favourable since it is indefinite and its resulting linear system of equations requires special efforts to be solved iteratively. By contrast, if potential field equations are used in the modelling, many common Krylov subspace iterative solvers can be directly applied to the solving of the resulting linear system. Current potential function approaches for the modelling of magnetotelluric data from the literature are limited to those only using one-dimensional background models (e.g., halfspace models) which cannot consider exact two-dimensional boundary surfaces, and hence will adversely introduce unreal terrain-caused distortions to the magnetotelluric data. In this study, we have developed a potential function-based modelling approach that can handle arbitrarily complex two-dimensional boundary surfaces for marine magnetotelluric data modelling. We demonstrate the effectiveness and accuracy of this new approach with a realistic marine conductivity model.

Keywords: electromagnetic induction, magnetotelluric, potential function, modelling

INTRODUCTION

In magnetotelluric (MT) data modelling, one can choose to solve the Helmholtz equation for the electric field $\mathbf{E}$:

$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu\sigma\mathbf{E} = \mathbf{0}, \tag{1}$$

which describes the diffusive behaviour of the electromagnetic (EM) field in a conductive medium (Harrington, 2001). Here, $\omega = 2\pi f$ is the angular frequency, μ is the material's magnetic permeability which is assumed to be that of free space (μ_0) and does not change in space. σ is the conductivity distribution of the underlying Earth model, and i is the imaginary unit.

Using the potential function relation of $\mathbf{E} := -i\omega\mathbf{A} - \nabla\phi$ (and $\mathbf{H} := \mu^{-1}\nabla \times \mathbf{A}$, $\mathbf{H}$ is the magnetic field) where $(\mathbf{A}, \phi)$ are the vector and scalar potential functions, respectively, we obtain the Helmholtz equation for the vector potential function:

$$\nabla \times \nabla \times \mathbf{A} + i\omega\mu\sigma\mathbf{A} + \sigma\mu\nabla\phi = \mathbf{0}. \tag{2}$$

Taking the divergence of Equation 2 leads to

$$\nabla \cdot (i\omega\mu\sigma\mathbf{A} + \sigma\mu\nabla\phi) = 0. \tag{3}$$

Equations 2 and 3 are called the potential function equations for the three-dimensional (3-D) modelling of EM data (Badea et al, 2001).

Using the potential function approach (solving Equations 2 and 3) offers some unique advantages: (1) there is much less effort required when the resulting linear system is solved iteratively since its more well-posed, which is in contrast to iteratively solving the $\mathbf{E}$ field Helmholtz equation (Smith, 1996;

Farquharson and Miensopust, 2011; Grayver and Kolev, 2015); (2) additional insights of induction physics, e.g., galvanic EM inductions, can be gained which are useful to understand MT data. Since our marine models include coastlines and are often of continental scale (see Figure 1), the linear system in the forward modelling is often very large (at least of a few millions order),

METHODS AND RESULTS

Since the boundary potential function values cannot be directly obtained from the electric and magnetic fields, they will need to be properly calculated over any 2D boundary surface of the 3D Earth model of interest before the 3D MT data modelling is carried out. We have thoroughly derived the mathematical equations and their boundary conditions for doing so for which the details are not presented here due to the length limitation but are presented in Long and Wang (2024).

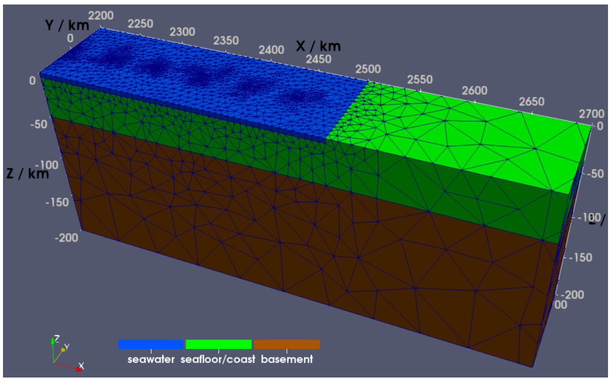

Figure 1: Localized view of a 3D tetrahedral mesh of the marine conductivity model. The vertical coastline is at x = 2500 km. Vertical boundary surfaces in the $x - z$ plane of this model are 2D surfaces.

We use the marine conductivity model over the Mendocino Fracture Zone (Wannamaker et al, 1989; Kim and Wessel, 2016; Materna et al, 2018) here as the example to demonstrate the capability and effectiveness of the newly developed potential function approach. To build a 3D tetrahedral mesh that is of good quality for the marine MT data modelling, we first build a 3D triangulated surface that represent the real bathymetry over the fracture area. This is done by interpolating the elevations of a 2D triangular surface from the grid of the real bathymetry (see Figure 3 (a)). Once the 3D triangulated surface is generated (see Figure 3 (c)), it is embedded into a volumetric space that contains proper air region, seawater and the basement region. In this example, we have used a local region of the bathymetry grid in Figure 3 (a-b) which has the designed ocean bottom MT sites (Figure 3 for details) as the central area. The 3D tetrahedral mesh used in this modelling extends 225 km in the east and west directions and 300 km in the south and north directions with the 20 MT sites as the centre. This results in a computational domain of $-225 <= x <= 225$ km, $-300 <= y <= 300$ km and $-50 <= z <= 30$ km in our relative Cartesian coordinate system in which the profile of the MT sites is now at approximately $x = 0$ (i.e., Easting at 0 km). The final tetrahedral mesh looks like the one in Figure 1 but without the coastal part.

The boundary surfaces of this marine model consist of four different 2D conductivity models. On each of them, a separate 2D MT modelling was carried out to find the boundary values of the degrees of freedom of the 3D tetrahedral mesh.

To demonstrate the modelling effectiveness, ten frequencies between 10^{-4} Hz and 0.1 Hz were used. The 3D solution was obtained using the generalized minimal residual (GMRES) iterative method. For the high frequencies ($>=$ 0.01 Hz), there was the concern of having insufficient mesh sizes and a high-order finite element method algorithm was used.

The computed 3D MT responses are shown in Figure 2. The basement conductivity is 100 Ωm. The seawater conductivity is 3.3 S/m. It is seen that the off-diagonal components of the impedance have the apparent resistivity and phase curves close to that of the 1D model but with noticeable differences. The deviations are due to bathymetry effects.

CONCLUSIONS

For complex Earth models that often have 2D boundary surfaces, it is found that existing potential function approach, despite its advantage of being solved efficiently with iterative solvers, cannot handle such scenarios. We therefore have developed this new, more complete potential function approach that can provide 2D boundary potential function values. The tested example over a real bathymetry model shows the approach is feasible and its modelling is accurate for complex terrain effects.

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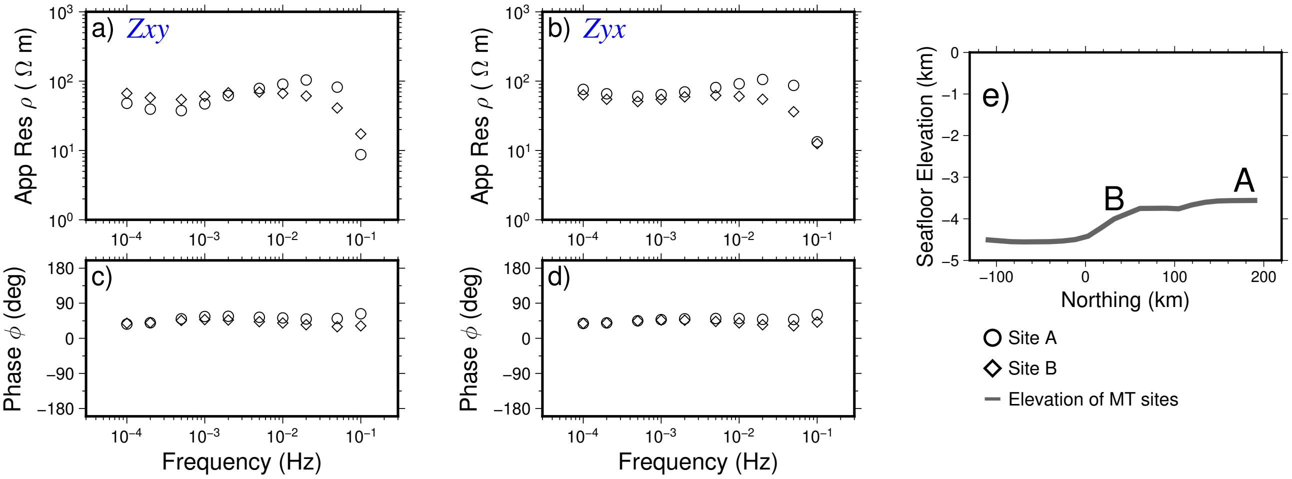

Figure 2: Impedance responses (only Z_{xy} and Z_{yx} components) of two sites (A and B) for the marine conductivity model with realistic bathymetry shown in Figure 3 (c). Panel a and b are the apparent resistivity curves; panel c and d are the phase curves. Panel e shows the elevation changes for all 20 MT sites that are approximately inline along the profile $x = 1956$ km in Figure 3. Note that Panel e uses the local coordinate system in which the exact locations of the sites A and B are: $y_A = 178$ km; $y_B = 32$ km.

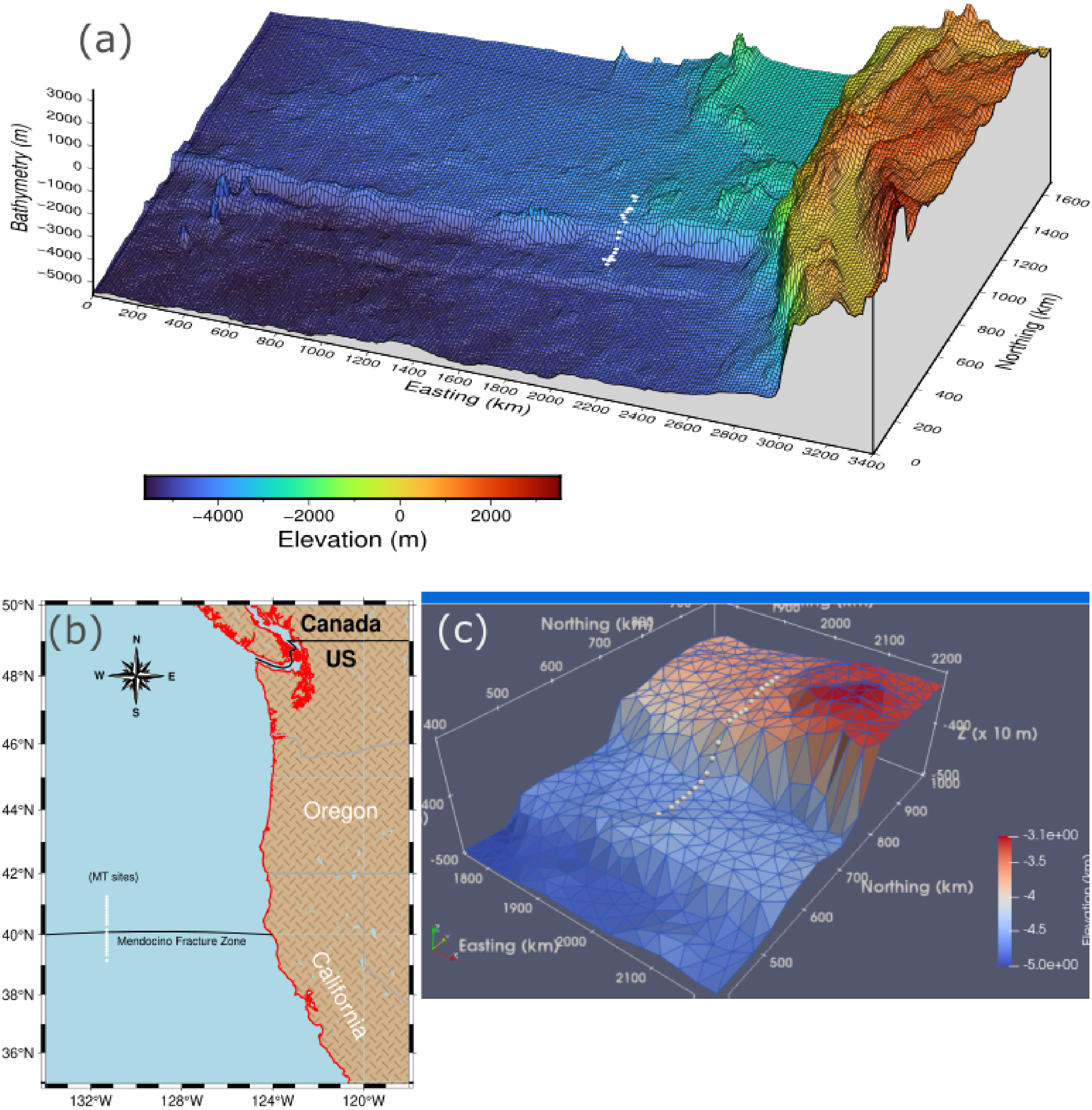

Figure 3: Marine Earth model with realistic bathymetry over the Mendocino Fracture Zone. Panel (a): the smoothed bathymetry/coast grid (longitude from 149° W to 118° W, latitude from 34° N to 47° N, which spans over 3400 km in the easting and 1600 km in the northing) from which triangulated meshes representing the bathymetry are built; Panel (b): Regional map showing the designed marine MT site locations in relative to the North America continent; Panel (c): the final triangulated regional bathymetry mesh required to build tetrahedral meshes for the 3D MT data modelling. In all panels, the seafloor MT site locations are indicated by the white dots which are approximately aligned with the line at Easting $(x) = 1956$ km and extend along the y direction (i.e., Northing) by 303 km. The MT sites are roughly equispaced along the profile.