4D Inversion Method Using Structural Coupling

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SUMMARY

Four-dimensional analysis using electromagnetic data is useful for monitoring volcanoes threatened by phreatic eruptions. In the past, 4D analysis has been performed by taking the difference of models estimated at each observation time, but its estimation accuracy and computational cost have been problematic. In this study, we propose a method that considers 4D analysis in the framework of joint inversion and imposes structural coupling between models at each observation time. The results of the synthetic test using magnetic inversion analysis show that changes in the model were concentrated closer to input model changes than in the conventional method of individual inversion.

Keywords: 4D Inversion, Joint Inversion, Structural Coupling

INTRODUCTION

Phreatomagmatic eruptions are thought to occur when a high-temperature fluid rapidly vaporizes and expands in a hydrothermal system that exists in the shallow subsurface for some reason. The mechanisms behind this phenomenon remain largely unknown, and elucidating them requires accurate monitoring of the spatiotemporal distribution of subsurface high temperature volcanic fluid. Consequently, as a preliminary step to monitoring, numerous attempts have been made to estimate the spatial distribution of fluids through electromagnetic inversion using electromagnetic field data, which are highly sensitive to heat and fluid.

In recent years, as the next step towards monitoring, 4D analysis has been conducted to estimate spatiotemporal changes. The conventional method used in this context involves estimating models at each observation time and determining spatial and temporal changes by taking the differences between these models (e.g., Minami et al., 2018; Mannenn et al., 2019).

However, there are two significant issues with this conventional 4D analysis method. First, it can detect apparent spatiotemporal changes caused by factors such as the configuration of observation points and data quality (Kim et al., 2005). This can lead to erroneous interpretations, making it unreliable for monitoring. Second, the computation time is exceedingly long. Since the conventional method requires model estimation at each observation time, including parameter exploration, detecting spatiotemporal changes can take several weeks to months. This poses a major obstacle to the timely monitoring of subsurface fluids.

To address these issues, this study proposes an algorithm that considers 4D analysis as a joint inversion and imposes structural coupling between models at each observation time. For the structural coupling method, we use Group Lasso (Yuan et al., 2005; Utsugi, submitted), which can impose structural coupling and sparseness simultaneously. The introduction of this method is expected to reduce apparent spatiotemporal changes and improve the accuracy of extracting actual spatiotemporal changes. Additionally, estimating models for multiple time points in a single calculation is anticipated to contribute to a reduction in computation time.

METHODS

Let the observation times be denoted as T_1, $T_2, \dots, T_n$, and the data observed at each time as $\boldsymbol{y}_{T_1}, \boldsymbol{y}_{T_2}, \dots, \boldsymbol{y}_{T_n}$, with the three-dimensional estimation model represented by $\boldsymbol{\beta}_{T_1}, \boldsymbol{\beta}_{T_2}, \dots, \boldsymbol{\beta}_{T_n}$. In this case, the observation equation can be summarized using the four-dimensional vector $\boldsymbol{\beta} = [\boldsymbol{\beta}_{T_1}, \boldsymbol{\beta}_{T_2}, \dots, \boldsymbol{\beta}_{T_n}]$.

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{\beta}) \tag{1}$$

Here, $\boldsymbol{y}=[\boldsymbol{y}_{T_1},\ \boldsymbol{y}_{T_2},\dots,\boldsymbol{y}_{T_n}]$ is the data vector, and $\boldsymbol{f}$ is the kernel functions that satisfy the observation equations at each observation time.

In this study, we consider equation (1) as a joint inversion of multiple problems and propose introducing structural coupling between the models at each observation time. Structural coupling is a constraint commonly used in joint inversions that combine different physical models (for example, magnetic structure with density structure, resistivity structure with seismic wave velocity structure, etc.), imposing a high correlation between each physical model. Specifically, established methods such as cross-gradient (Meju and Gallardo, 2004) and fuzzy c-means clustering (Pasche and Tronicke, 2007) are used, allowing each physical model to exhibit high correlation while permitting structural differences to partially reproduce each dataset.

Therefore, by applying this method to the estimation of spatiotemporal changes, the models at each observation time become highly correlated, such that temporally invariant structures will have the same phase at each time, suppressing the detection of apparent changes and enabling the detection of structural differences, i.e., spatiotemporal changes. Additionally, this method allows for the estimation of models at multiple times with a single calculation. Thus, compared to the traditional approach, which required calculations for each observation time, a reduction in computation time is expected.

In this study, we use Group Lasso (Yuan et al., 2005) to impose this structural coupling. Group Lasso is a type of sparse regularization method that has the efficacy of selecting explanatory variables by groups specified by the analyst. Therefore, by grouping the values at the same location among the models at each observation time, it is expected that structural coupling and sparsity can be introduced simultaneously.

Utsugi (2024, submitted) reported that using the group lasso penalty alone is likely to derive an overly concentrated and unrealistic model, and to compensate for this drawback, they used the combined penalty of the group lasso and its competitive L2 norm penalty. Therefore, in this study, we combine Group Lasso with the L2 norm (allowing for some extent of spread) as constraints on the model vector. The problem to be solved then becomes as follows:

$$\min.\ \frac{1}{2}\|\boldsymbol{y}-\boldsymbol{f}(\boldsymbol{\beta})\|_2^2+\lambda_1\sum\left\|\boldsymbol{\beta}_{\mathscr{g}_j}\right\|+\frac{\lambda_2}{2}\|\boldsymbol{\beta}\|_2^2 \qquad (2)$$

$$\boldsymbol{\beta}_{\mathscr{g}_j}=[\beta_{T_1,j},\beta_{T_2,j},\dots,\beta_{T_n,j}] \qquad (j=1,2,\dots,m)$$

where λ_1 and λ_2 are hyper parameters, and $\beta_{T_i,j}$ is j-th component of $\boldsymbol{\beta}_{T_i}$. The second term in eq. (2), $\sum\left\|\boldsymbol{\beta}_{\mathscr{g}_j}\right\|$, is the group lasso penalty. $\left\|\boldsymbol{\beta}_{\mathscr{g}_j}\right\|$ denotes the Euclidean norm, and $\boldsymbol{\beta}_{\mathscr{g}_j}$ is a subvector consisting of model elements belonging to the j-th group that imposed the structural coupling. In our study, the model elements of each grid cell subdividing the model space are grouped on the time sequence $T_1,\dots,T_n$. Thus, $\left\|\boldsymbol{\beta}_{\mathscr{g}_j}\right\|$ is explicitly written as

$$\sum_j\left\|\boldsymbol{\beta}_{\mathscr{g}_j}\right\|=\sum_j^M\sqrt{\beta_{T_1,j}^2+\beta_{T_2,j}^2+\cdots+\beta_{T_n,j}^2}$$

where M is the total number of subsurface grid cells.

SYNTHETIC TESTS

We tested our method using synthetic model with magnetic inversion analysis. Figure 1 shows the cross-sections at x = 0 km of the magnetic models of 3 observation times, where a Cartesian coordinate system (x, y, z) was introduced in the subsurface model space, and x, y, and z-axes directed northward, eastward, and downward, respectively. We set the model area of 4 km×4 km × 2 km size divided by 32000 cubic cells. Observation points are set at the height of 50 m. As time passes, the area of low magnetization extends to the basement on the west side. The inclination and declination of the geomagnetic field were assumed to be 50° and -7°, respectively, and the direction of the magnetization vector of each block was parallel to that of the geomagnetic field. We use the magnetic anomaly generated by the input model plus 2% noise as the input data.

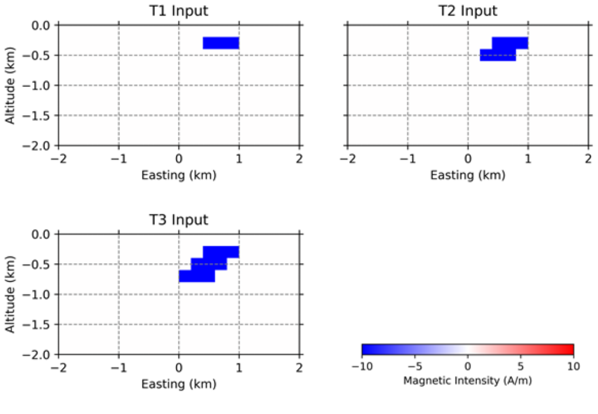

Figure 1. Cross-sections at x = 0 km of the input magnetic models at 3 observation times. As time passes, the area of low magnetization extends to the basement on the west side.

The Figure 2 shows the cross-sections at x = 0 km of the results of synthetic test. The left column shows the time change from T_1 to T_2 and the right column shows the time change from T_2 to T_3. Figure 2a and Figure 2b show the input model. Figure 2c and Figure 2d represent the result of 4d inversion of this study, and Figure 2e and Figure 2f represent the result of individual inversion (Utsugi, 2019) at each observation time. Figure 2c and Figure 2d show that the method proposed in this study is sufficient to detect the change parts of the magnetic model. Furthermore, the change parts appear closer to the input model than the areas where inversions were performed individually (Figure 2e and Figure 2f). This indicates that our method works well for detecting spatiotemporal changes.

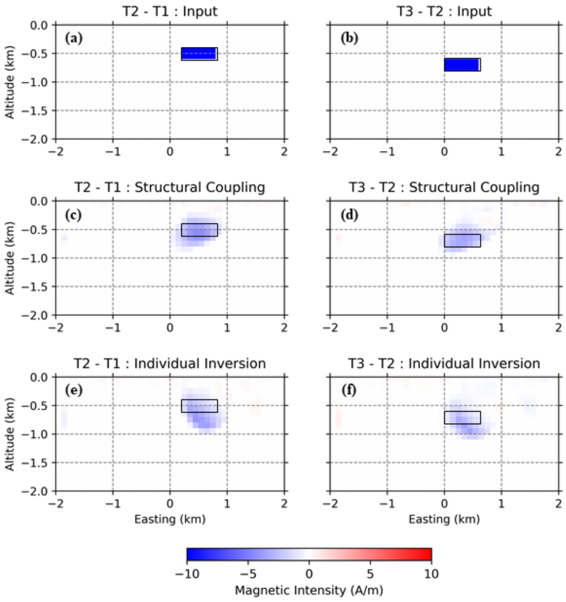

Figure 2. Cross-sections at x = 0 km of the result of the synthetic test. The left column shows the time change from T_1 to T_2 and the right column shows the time change from T_2 to T_3. (a) and (b) show the input model. (c) and (d) represent the result of 4d inversion of this study, and (e) and (f) represent the result of individual inversion (Utsugi, 2019) at each observation time.

DISCUSSION

The method is applicable to other geophysical methods, including magnetotelluric (MT) inversion analysis. We plan to apply the method to MT inversion analysis by the time of this presentation.

In addition, since the location and magnitude of the spatiotemporal changes detected by the two hyper-parameters λ_1 and λ_2 can vary, an objective method for determining them is needed. The idea of adjusting the RMS residuals of the model at each observation time has been pointed out in a previous study (Kim et al., 2012) as one possible metric.

Additionally, there are already some studies about 4d ERT inversion (e.g. Kim et al., 2009), so we should compare them with our method and see in which cases our method should be used especially.

This study uses the Group Lasso as a method of imposing structural coupling between models at each observation time, but other methods are possible. For example, it will be possible to impose a constraint similar to some previous methods by constraining the difference between the models at each observation time. This is an issue to consider in the future.

After a thorough study of the methodology has been completed, we would like to apply it to real data. We consider Mt. Kuju Iwo-yama at Oita prefecture, in southwestern Japan, as a site for application. This mountain has a well-developed subsurface hydrothermal system, and a phreatic eruption occurred there in 1995. Applying the methodology of this study to such a site would allow us to test the feasibility of monitoring phreatomagmatic eruption sites.

CONCLUSIONS

4D analysis using electromagnetic data is useful for monitoring volcanoes where phreatic eruptions can occur. In this study, we propose a method that considers 4D analysis within the framework of joint inversion and imposes structural coupling between models at each observation time, thereby achieving both estimation accuracy and computational cost. The results of synthetic tests on magnetic inversions show that changes in the model are concentrated closer to the input than in conventional methods, suggesting the usefulness of the proposed method. In the presentation, we will show the results of the application to MT inversion.

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REFERENCES

Kim, J. H. (2005) Four dimensional inversion of DC resistivity monitoring data. In Near Surface 2005-11th European Meeting of Environmental and Engineering Geophysics. European Association of Geoscientists & Engineers. doi: 10.3997/2214-4609-pdb.13.A006

Kim, J. H., Yi, M. J., Park, S. G., & Kim, J. G. (2009). 4-D inversion of DC resistivity monitoring data acquired over a dynamically changing earth model. Journal of Applied Geophysics, 68(4), 522-532.
doi: 10.1016/j.jappgeo.2009.03.002

Kim, J. H., Tsourlos, P.I., & Supper, R. (2012) 4D inversion of L1 and L2 norm minimizations. Berichte Geol. B.-A., 93, ISSN 1017 - 8880 Time Lapse Data Inversion, 106-113.

Paasche, H., & Tronicke, J. (2007) Cooperative inversion of 2D geophysical data sets: A zonal approach based on fuzzy c-means cluster analysis. Geophysics, 72(3), A35-A39.
doi: 10.1190/1.2670341

Mannen, K. et al., (2019) Source constraints for the 2015 phreatic eruption of Hakone volcano, Japan, based on geological analysis and resistivity structure. Earth, Planets and Space, 71, 1-20.
doi: 10.1186/s40623-019-1116-5

Gallardo LA, Meju MA (2003) Characterization of heterogeneous near-surface materials by joint 2D inversion of dc resistivity and seismic data. Geophys Res Lett 30:1658.
doi:10.1029/2003GL017370

Utsugi, M. (2019) 3-D inversion of magnetic data based on the L1–L2 norm regularization. Earth, Planets and Space, 71, 1-19.
doi: 10.1186/s40623-019-1052-4

Yuan, M., & Lin, Y. (2006) Model selection and estimation in regression with grouped variables. Journal of the Royal Statistical Society Series B: Statistical Methodology, 68(1), 49-67.
doi: 10.1111/j.1467-9868.2005.00532.x

Minami, T., Utsugi, M., Utada, H., Kagiyama, T., & Inoue, H. (2018) Temporal variation in the resistivity structure of the first Nakadake crater, Aso volcano, Japan, during the magmatic ruptions from November 2014 to May 2015, as inferred by the ACTIVE electromagnetic monitoring system. *Earth, Planets and Space*, *70*, 1-10.
doi: 10.1186/s40623-018-0909-2