3D magnetotelluric inversion with structurally guided regularization constraint

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SUMMARY

We propose a technique for the structurally regularized three-dimensional magnetotelluric inversion scheme. In the proposed method, the structural resemblance between the inverted electrical resistivity model and the independently derived guiding model is enforced by controlling weights of the roughness operator for regularization. Using a simple method, we forced the weights to be small across presumed discontinuities in the guiding model, allowing for sharp changes in resistivity values across the discontinuity. We forced the weights to be large along blocks with similar values in guiding physical parameters, enforcing smooth changes of resistivity along the blocks. Numerical inversion tests suggest that our guided regularization inversion results can perform better than conventional smooth inversion. In the field data application, the guided inversion model is more consistent with the geophysical and geological inferences, while data fitting is as well as that of the conventional smooth inversion model. It is important to note that the proposed method does not require an additional term in the objective function. Only the trade-off between data misfit and a single regularization term is considered, as in a standard regularized MT inversion, thus avoiding the issue of simultaneous selection of multiple trade-off parameters in the objective function.

Keywords: integrated geophysical imaging, guided magnetotelluric inversion

INTRODUCTION

The integration of one or more independently derived physical Earth models in the magnetotelluric (MT) inversion can result in a more restricted solution space of electrical resistivity models, leading to higher confidence for geological interpretations. The resulting resistivity model is required to explain the observation and satisfy a certain relationship to the guiding models. One common assumption regarding the correlation between various geophysical methods is that they sense similar geological structures, particularly their boundaries. In this regard, a spatial correlation between changes in electrical resistivity and other physical parameters can constrain the MT inversion without prescribing any direct relationship between the parameters. This way of inversion, known as structurally constrained MT inversion, results in a resistivity model that resembles the guiding model.

This study proposes a scheme for structure-guided three-dimensional (3-D) MT inversion by assigning weights to the regularization matrix of the inversion based on the spatial variation of the guiding model. The weights are small across boundaries presumed by the guiding model, allowing a sharp contrast of resistivity locally. Unlike the widely used cross-gradient coupling method (Gallardo and Meju, 2003), the proposed method does not require an additional term in the objective function. Only the trade-off between data misfit and regularization terms is considered, as in a standard regularized MT inversion. Thus, traditional and well-justified ways, such as the L-curve and the ABIC, can be used to determine the optimum balance between the terms in the objective function. This is an advantage of our approach over the widely used cross-gradient method, in which an additional coupling term is often necessary in practice.

METHODOLOGY

Regularized MT inversion seeks a model m that minimizes the following objective function:

$$\Phi(\boldsymbol{m}) = \|\boldsymbol{d} - \boldsymbol{F}(\boldsymbol{m})\| + \alpha^2\|\boldsymbol{R}\boldsymbol{m}\|^2 . \quad (1)$$

The first and second terms on the right-hand side correspond to the data misfit and the regularization, respectively. The multiplier α^2 serves as a trade-off parameter for finding an optimum balance between the data misfit and regularization terms.

The form of the regularization term is highly flexible, but it is typically chosen to measure the spatial roughness of the model. Spatial roughness is associated with derivatives of the model function. For instance, a first-derivative operator of the model function, which measures the simple arithmetic differences of the model parameter of adjacent blocks, can be expressed as

$$\|\boldsymbol{Rm}\|^2 = \left\| \begin{matrix} m_1^I - m_1^{II} \\ \vdots \\ m_i^I - m_i^{II} \\ \vdots \\ m_N^I - m_N^{II} \end{matrix} \right\|^2 \tag{2}$$

where m_i^I and m_i^{II} are a pair of model parameters (the common logarithm of the subsurface resistivity) sharing the i-th face (see Figure 1), and N is the total number of faces in the subsurface except for the side and bottom boundaries. This form of regularization results in a model with a minimum spatial gradient.

To impose structural constraint into the MT inversion, we propose a simple modification to Eq. (2) that involves a weighting value to the difference between adjacent model parameters. The weights are determined based on the guiding model, which is an independently derived 3-D model from which structural information is referred for the resistivity inversion. For example, the guiding model can be a model of other physical parameters, such as velocity or density structure, that is assumed to be structurally correlated with the electrical resistivity.

The modified regularization term with the weighted difference operator can be expressed as:

$$\|\boldsymbol{Rm}\|^2 = \left\| \begin{matrix} w_1(m_1^I - m_1^{II}) \\ \vdots \\ w_i(m_i^I - m_i^{II}) \\ \vdots \\ w_N(m_N^I - m_N^{II}) \end{matrix} \right\|^2 \tag{3}$$

where the weight w_i is proposed to be of the following form:

$$w_i = 1/\exp(\eta|v_i^I - v_i^{II}|)\,. \tag{4}$$

v_i^I and v_i^{II} is entries in the guiding model located precisely at the center of the m_i^I and m_i^{II} blocks, respectively (see Figure 1). The weight is inversely proportional to the difference of two adjacent values in the guiding model (v_i^I and v_i^{II}). Any difference between v_i^I and v_i^{II} reduces w_i. When Eq. (3) is minimized, smaller weights allow higher contrast between adjacent resistivity blocks in the inverted model and vice versa.

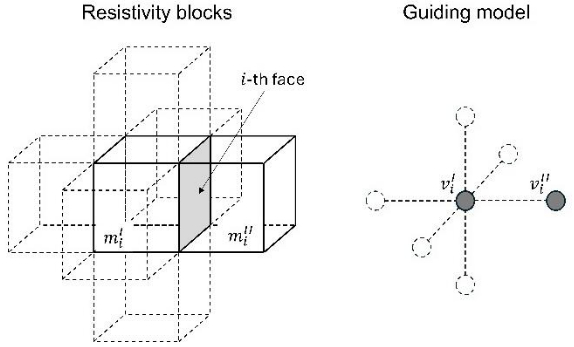

Figure 1. Illustration of resistivity blocks in the inversion mesh (left) and the associated values in the guiding model (right).

RESULTS

We used a synthetic model (Figure 2) that is also examined in Siripunvaraporn et al. (2005) and Usui et al. (2017). The inverted data was full-component MT impedance tensors at ten periods from 1 to 1000 s. Gaussian random noises were added with a standard deviation of 3 per cent. We implemented our method by modifying the regularization constraint of the FEMTIC inversion code (Usui, 2015). The Earth was discretized into 70 x 70 x 69 hexahedral blocks. The starting model of the inversion was a uniform 100 Ωm Earth.

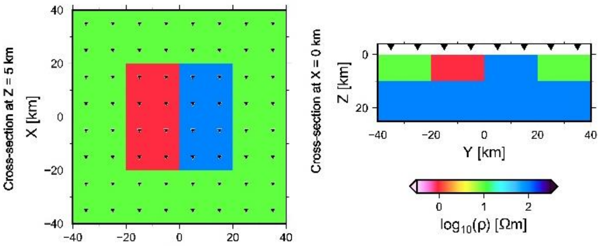

Figure 2. True resistivity model. The left figure is a horizontal cross-section at 5 km depth, and the right is a vertical cross-section at X = 0 km. Inverted triangles denote observation stations.

We used the logarithm of the true resistivity model as the guiding model. Then, we calculated the weights following Eq. (4), with $\eta = 2$. The guiding model and the corresponding regularization weights are shown in Figure 3. The weights are one within uniform structures. In the boundaries between 1 and 100 Ωm (red-blue interfaces), the weights are equal to 0.0183, whereas in the boundaries between 1 and 10 Ωm (red-green interfaces) and between 100 and 10 Ωm (blue-green interfaces), the weights are equal to 0.1353.

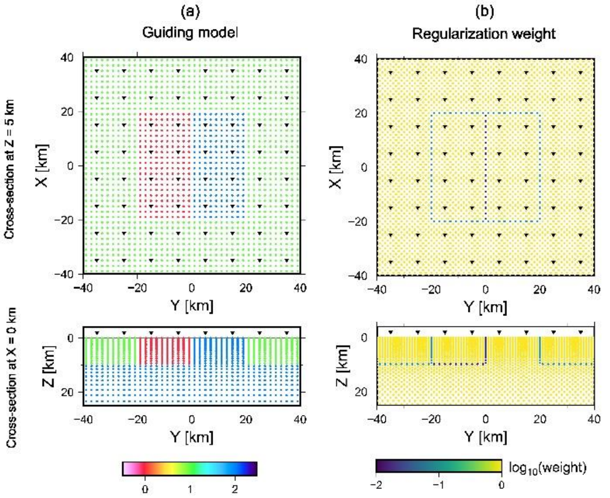

Figure 3. The guiding model (a) and the weights for the regularization (b) used for the guided inversion. The upper figures are horizontal cross-sections at 5 km depth, and the lower figures are vertical cross-sections at X = 0 km.

The final model resulting from inversion using the guided regularization method is shown in Figure 4, compared with the conventional smooth inversion result. The two results have an RMS data misfit of 1.00 and thus are statistically equivalent. The model misfits are 0.144 and 0.065 for the smooth and guided inversion models, respectively, indicating that the guided inversion result is closer to the true model. It is obvious that the resistivity value of the anomalies and the boundaries between different anomalies are resolved very accurately in the guided inversion model.

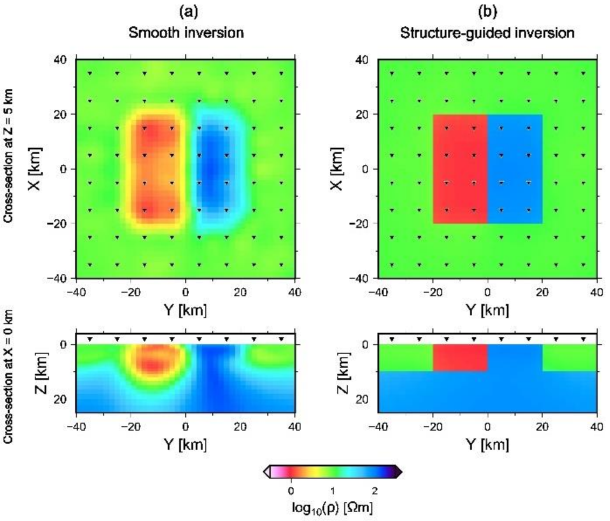

Figure 4. Inversion results using conventional smooth inversion (a) and the guided regularization inversion (b). The upper figures are horizontal cross-sections at 5 km depth, and the lower figures are vertical cross-sections at X = 0 km. Inverted triangles denote the observation stations. The true model of the inversion is shown in Figure 2.

More results will be discussed in the presentation, including the application of our guided inversion method to wide-band MT data in Southern Tohoku, Northeast Japan, with structural constraints from a seismic velocity structure. Data fitting of the guided inversion was comparable with that of smooth inversion. However, the guided inversion model is more consistent with geophysical and geological inferences. The vertical transition of structures around the depth of the crust-mantle boundary was detected. Low to high resistivity contrast across the seismically determined upper boundary of the subducting slab was more clearly imaged by the guided inversion.

CONCLUSION

We propose a structure-guided 3-D MT inversion scheme where the structural resemblance between the inverted resistivity model and the independently derived guiding model is enforced by controlling weights of the roughness operator for regularization. Numerical inversion tests suggest that our guided regularization result is closer to the true model than conventional smooth inversion result. In the field data application, the guided inversion model is more consistent with the geophysical and geological inferences. A practical advantage of the proposed method is that no additional coupling term in the objective function is necessary as the structural constraint is imposed on the regularization. Thus, the problem of simultaneous selection of multiple trade-off parameters in the objective function can be avoided.

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