Hilbert transform of the frequency normalized impedance data: Application to the dispersion relations in magnetotellurics

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SUMMARY

The real and imaginary parts of the Fourier transform of a causal signal can be obtained from each other by the Hilbert transform. The numerical computation can be carried out by the Fast Fourier Transform or the convolution of the data with an appropriate Hilbert kernel. However, magnetotelluric data are usually unequally sampled because the data points with less noise contribution are selected. A method and Python software are developed for the Hilbert transformation of unequally sampled data and applied to the real and imaginary part of the frequency normalized impedance to examine if the dispersion relations are satisfied for a given impedance tensor. The method involves the reconstruction of the measured data by a linear combination of an analytic interpolation function distributed along the horizontal axis. Then, the decomposition coefficients that provide a fit between the measured and the reconstructed data are solved by the linear least-squares method with singular value decomposition. Finally, the discrete Hilbert transformed data are constructed at any desired abscissa values by the sum of the analytic Hilbert transform of the interpolation function using the decomposition coefficients solved in the previous step. Examples from the magnetotelluric synthetic and field data, including out-of-quadrant phase cases, are given.

It is demonstrated that the dispersion relations of the frequency normalized impedance function are valid for all types of structures, including 1D, 2D and 3D cases. A set of theoretical data calculated for a 3D subsurface by ModEM software and corresponding dispersion curves computed with the suggested algorithm provided perfect fits. The dispersion relation for the FNI function is also valid for out-of-quadrant phase cases caused by 3D strong resistivity contrasts. However, electrical anisotropy and galvanic distortion could also be considered to explain the out-of-quadrant phase. Therefore, the validation of dispersion relations in such cases needs to be investigated.

Keywords: Magnetotellurics, Causal systems, Dispersion relations, Hilbert transform, Unequally sampled data.